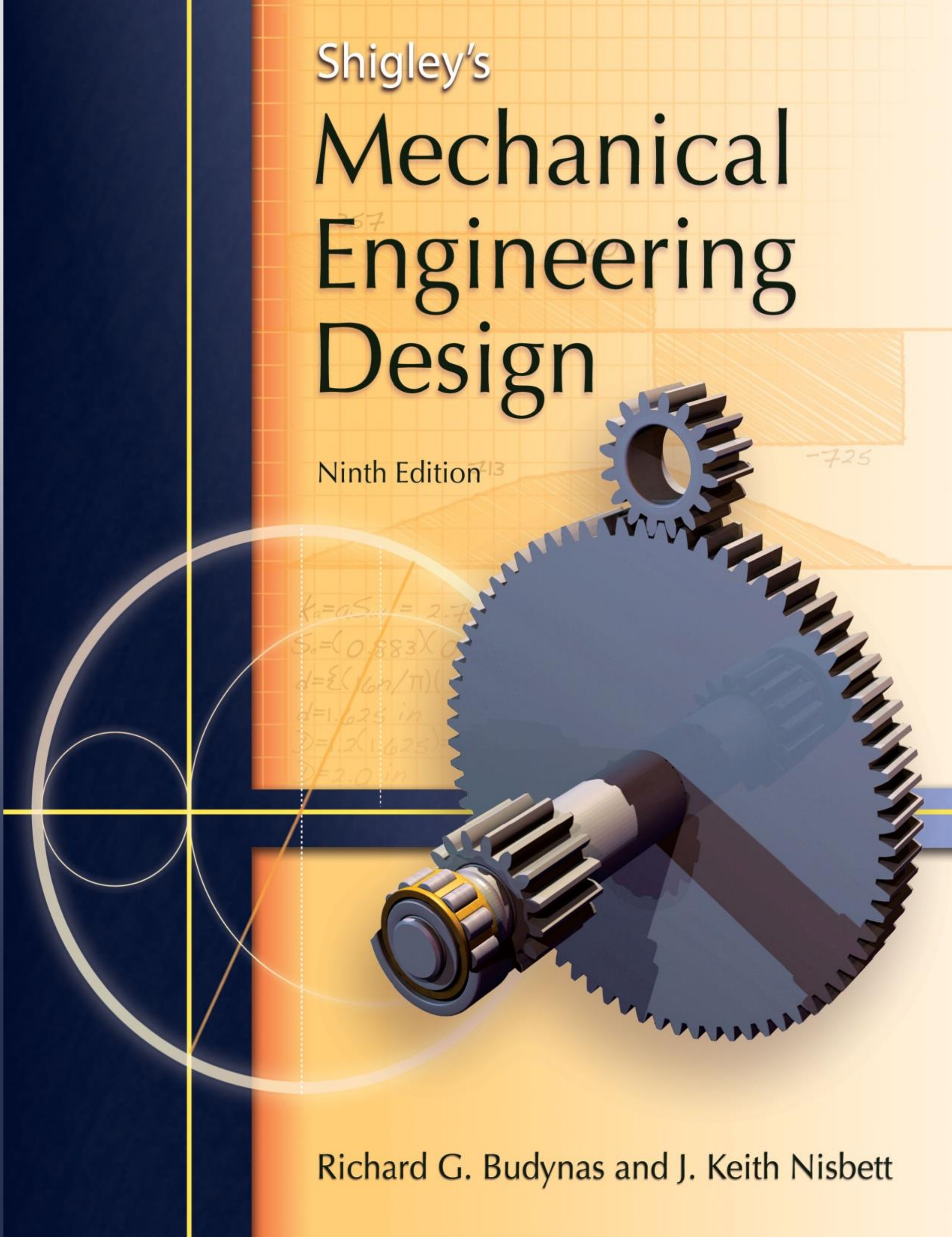
Lecture Slides

Chapter 17

Flexible Mechanical Elements



Chapter Outline

Characteristics of Some Common Belt Types

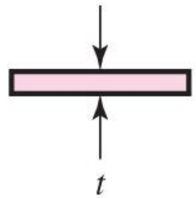
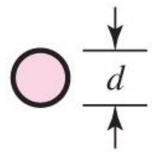
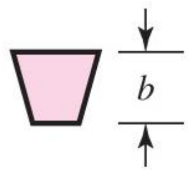
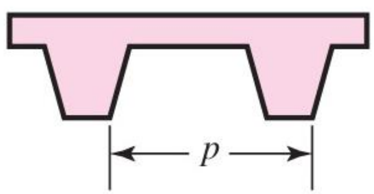
Belt Type	Figure	Joint	Size Range	Center Distance
Flat		Yes	$t = \begin{cases} 0.03 \text{ to } 0.20 \text{ in} \\ 0.75 \text{ to } 5 \text{ mm} \end{cases}$	No upper limit
Round		Yes	$d = \frac{1}{8}$ to $\frac{3}{4}$ in	No upper limit
V		None	$b = \begin{cases} 0.31 \text{ to } 0.91 \text{ in} \\ 8 \text{ to } 19 \text{ mm} \end{cases}$	Limited
Timing		None	$p = 2$ mm and up	Limited

Table 17–1

Flat-Belt Geometry – Open Belt

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2 - (D-d)^2} + \tfrac{1}{2}(D\theta_D + d\theta_d)$$

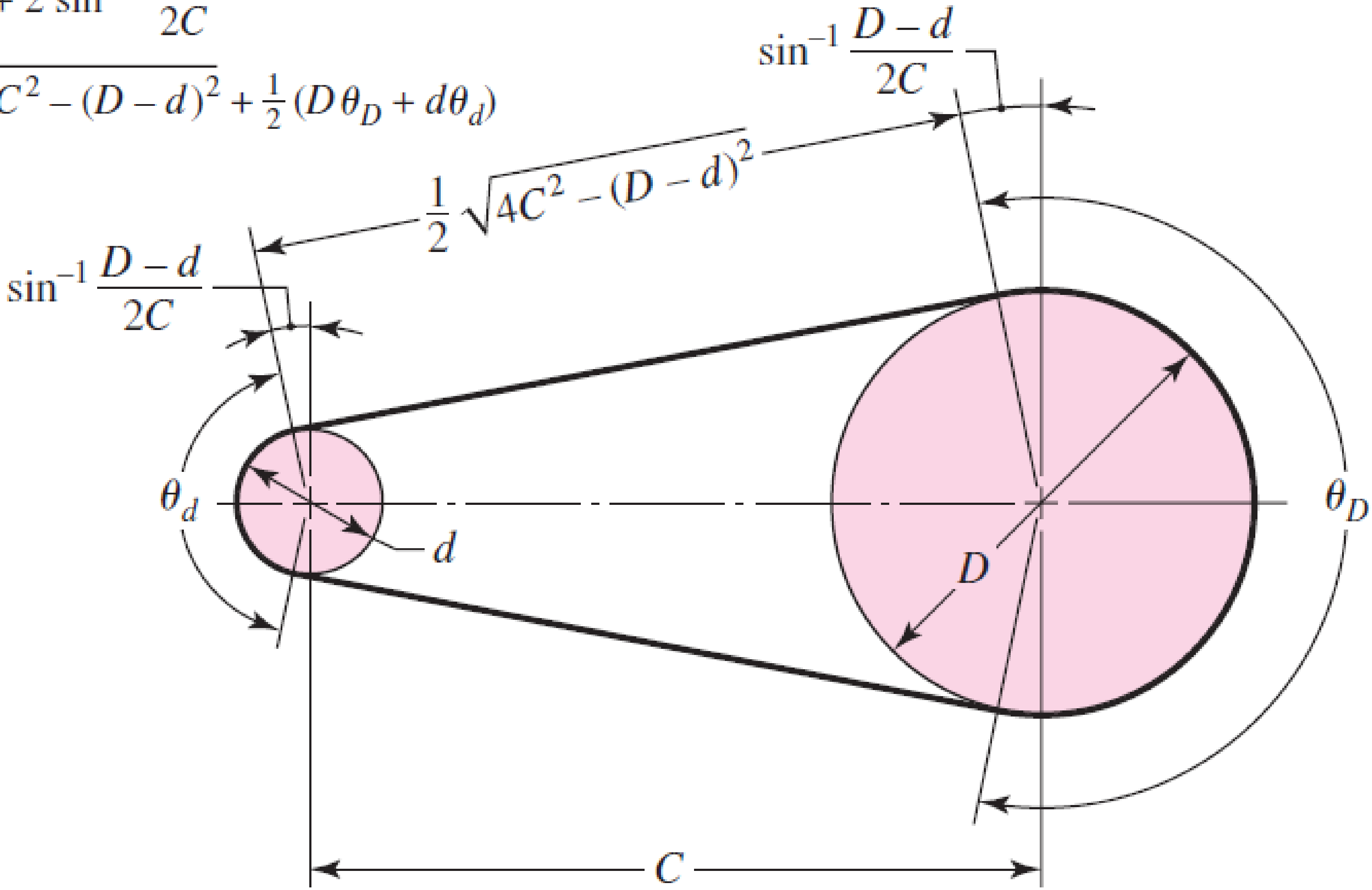


Fig.17–1*a*

Flat-Belt Geometry – Crossed Belt

$$\theta = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

$$L = \sqrt{4C^2 - (D+d)^2} + \frac{1}{2}(D+d)\theta$$

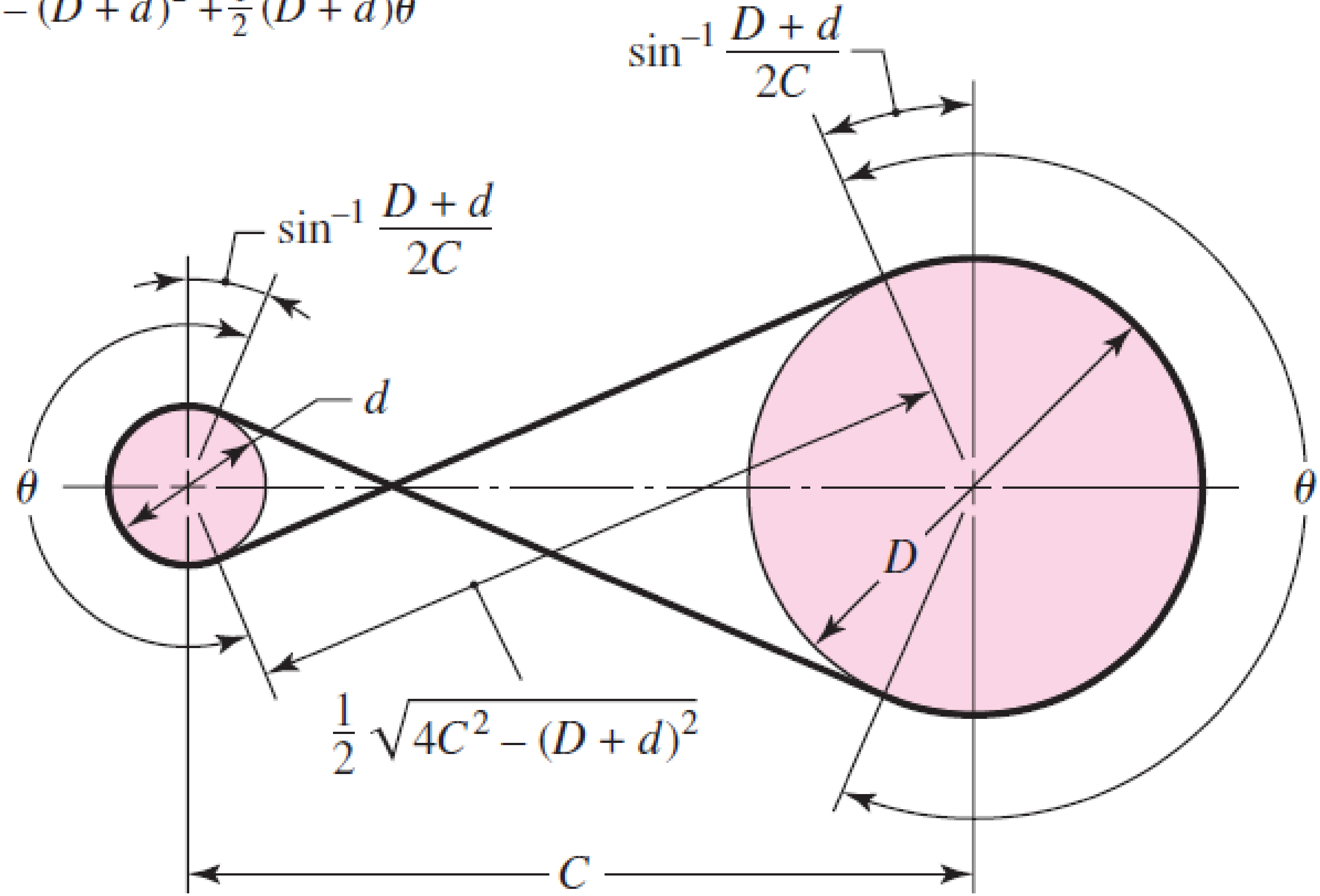
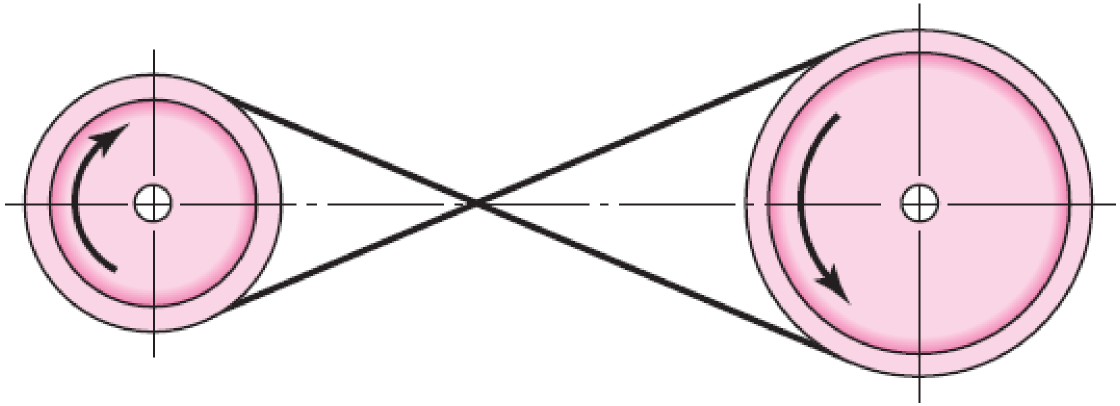
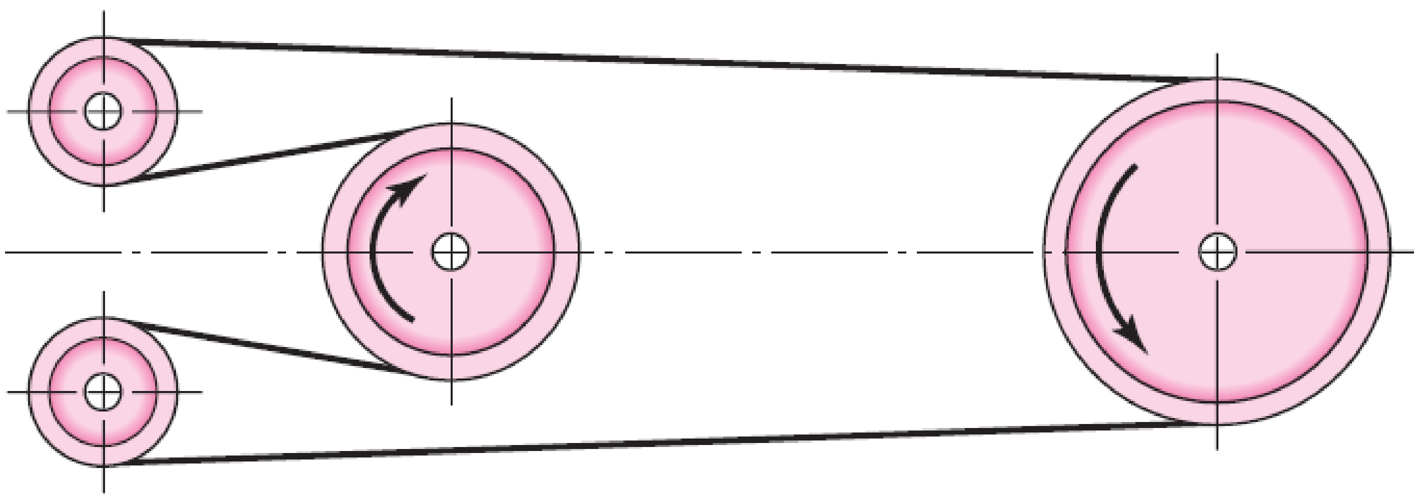


Fig.17–1*b*

Reversing Belts



(b)



(c)

Fig.17–2

Flat-belt with Out-of-plane Pulleys

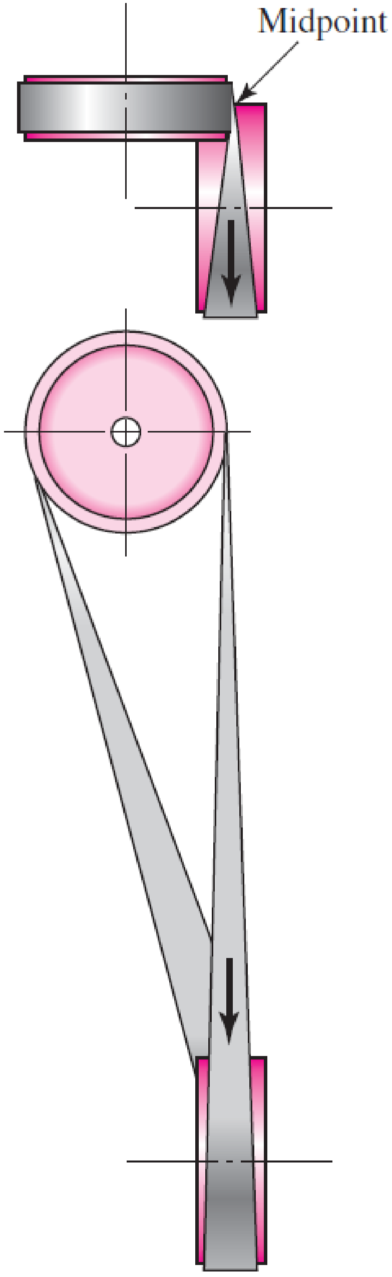


Fig.17–3

Flat-belt Shifting Without Clutch

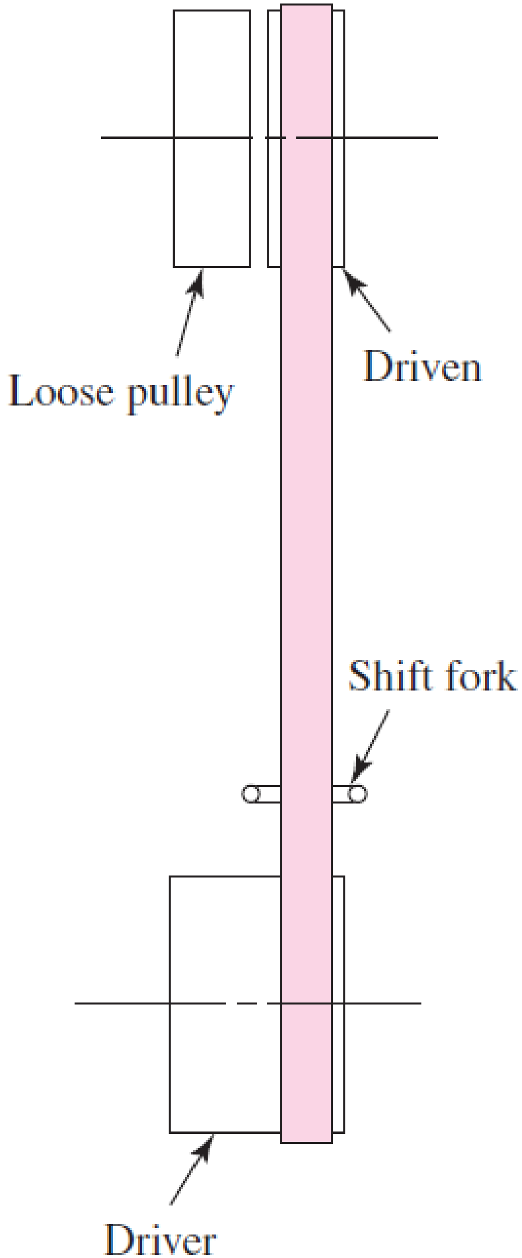
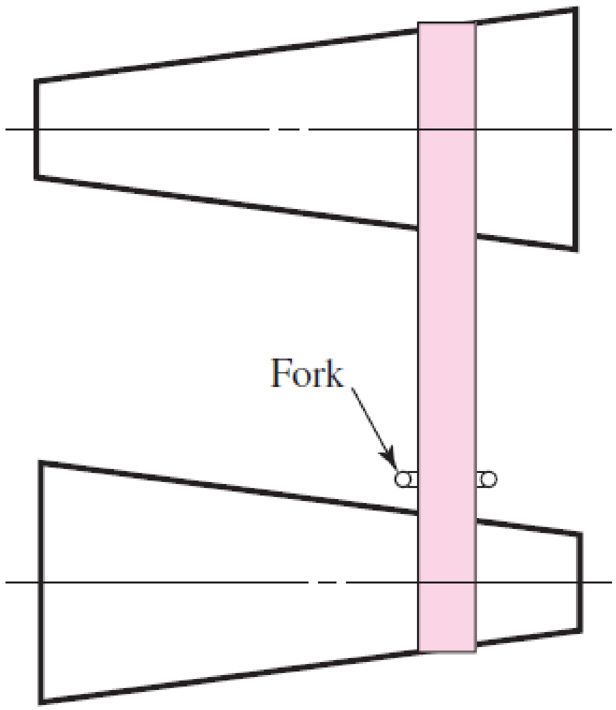


Fig.17–4

Variable-Speed Belt Drives



(a)

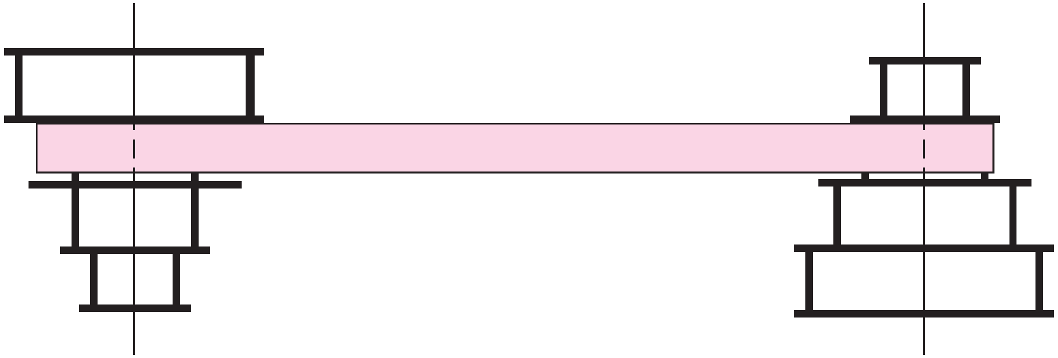


Fig.17–5 (b)

Free Body of Infinitesimal Element of Flat Belt

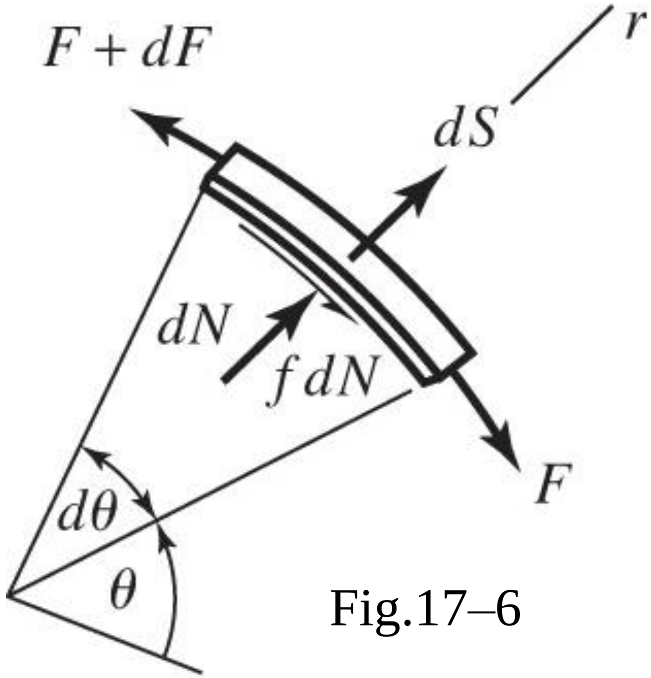


Fig.17–6

$$dS = (mr\,d\theta)r\omega^2 = mr^2\omega^2\,d\theta = mV^2\,d\theta = F_c\,d\theta \qquad \text{(a)}$$

$$\sum F_r = -(F + dF)\frac{d\theta}{2} - F\frac{d\theta}{2} + dN + dS = 0$$

$$dN = F\,d\theta - dS \qquad \text{(b)}$$

Free Body of Infinitesimal Element of Flat Belt

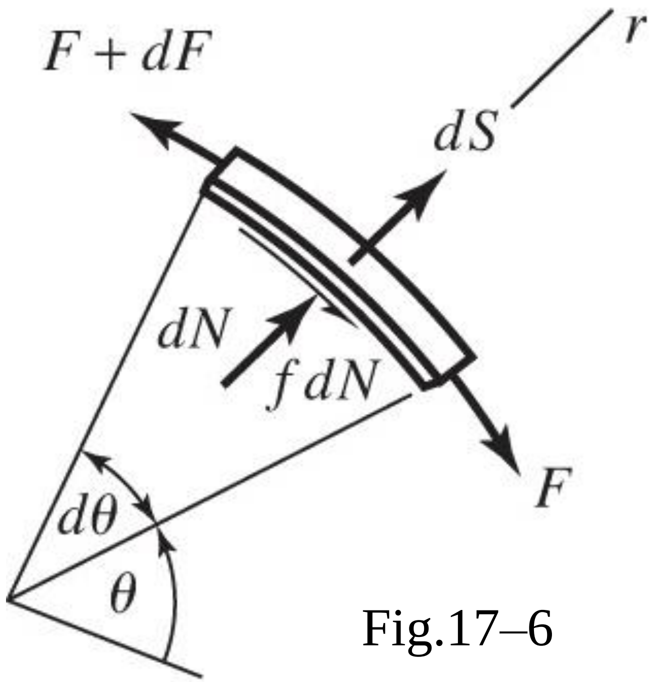


Fig.17–6

$$\sum F_t = -f\,dN - F + (F + dF) = 0$$

$$dF = f\,dN = f F\,d\theta - f\,dS = f F\,d\theta - f m r^2 \omega^2\,d\theta$$

$$\frac{dF}{d\theta} - f F = -f m r^2 \omega^2 \qquad (c)$$

Analysis of Flat Belt

$$\frac{dF}{d\theta} - fF = -fmr^2\omega^2 \qquad \text{(c)}$$

$$F = A\exp(f\theta) + mr^2\omega^2 \qquad \text{(d)}$$

$$F \text{ at } \theta = 0 \text{ equals } F_2 \text{ gives } A = F_2 - mr^2\omega^2$$

$$F = (F_2 - mr^2\omega^2)\exp(f\theta) + mr^2\omega^2 \qquad \text{(17–5)}$$

$$F|_{\theta=\phi} = F_1 = (F_2 - mr^2\omega^2)\exp(f\phi) + mr^2\omega^2 \qquad \text{(17–6)}$$

$$\frac{F_1 - mr^2\omega^2}{F_2 - mr^2\omega^2} = \frac{F_1 - F_c}{F_2 - F_c} = \exp(f\phi) \qquad \text{(17–7)}$$

$$F_c = mr^2\omega^2$$

$$F_1 - F_2 = (F_1 - F_c)\frac{\exp(f\phi) - 1}{\exp(f\phi)} \qquad \text{(17–8)}$$

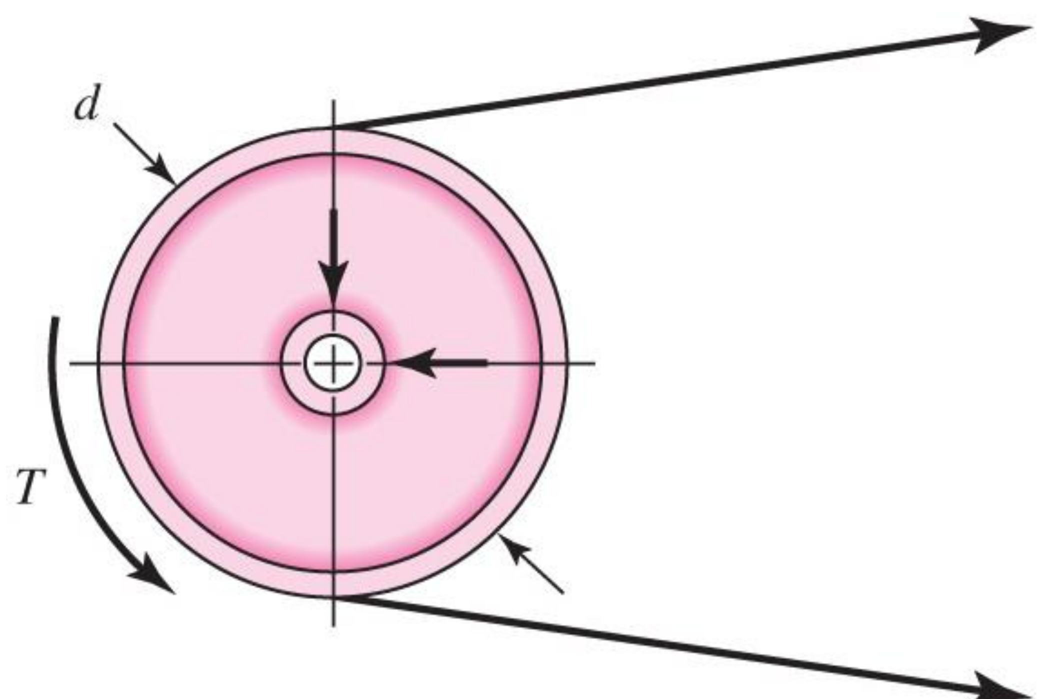
Hoop Tension Due to Centrifugal Force

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{w}{32.17}\left(\frac{V}{60}\right)^2 \qquad (e)$$

$w = 12\gamma bt$ lbf/ft where b and t are in inches

$V = \pi\, dn/12$ ft/min

Forces and Torques on a Pulley



$$F_1 = F_i + F_c + \Delta F/2 = F_i + F_c + \frac{T}{d}$$

$$F_2 = F_i + F_c - \Delta F/2 = F_i + F_c - \frac{T}{d}$$

Fig.17–7

F_i = initial tension

F_c = hoop tension due to centrifugal force

$\Delta F/2$ = tension due to the transmitted torque T

d = diameter of the pulley

Initial Tension

$$F_1 - F_2 = \frac{2T}{d} \qquad (h)$$

$$F_1 + F_2 = 2F_i + 2F_c$$

$$F_i = \frac{F_1 + F_2}{2} - F_c \qquad (i)$$

$$\frac{F_i}{T/d} = \frac{(F_1 + F_2)/2 - F_c}{(F_1 - F_2)/2} = \frac{F_1 + F_2 - 2F_c}{F_1 - F_2} = \frac{(F_1 - F_c) + (F_2 - F_c)}{(F_1 - F_c) - (F_2 - F_c)}$$

$$= \frac{(F_1 - F_c)/(F_2 - F_c) + 1}{(F_1 - F_c)/(F_2 - F_c) - 1} = \frac{\exp(f\phi) + 1}{\exp(f\phi) - 1}$$

$$F_i = \frac{T}{d}\,\frac{\exp(f\phi) + 1}{\exp(f\phi) - 1} \qquad (17\text{–}9)$$

Flat Belt Tensions

$$F_1 = F_i + F_c + \frac{T}{d} = F_c + F_i + F_i \frac{\exp(f\phi) - 1}{\exp(f\phi) + 1}$$

$$= F_c + \frac{F_i[\exp(f\phi) + 1] + F_i[\exp(f\phi) - 1]}{\exp(f\phi) + 1}$$

$$F_1 = F_c + F_i \frac{2\exp(f\phi)}{\exp(f\phi) + 1} \qquad (17\text{–}10)$$

$$F_2 = F_i + F_c - \frac{T}{d} = F_c + F_i - F_i \frac{\exp(f\phi) - 1}{\exp(f\phi) + 1}$$

$$= F_c + \frac{F_i[\exp(f\phi) + 1] - F_i[\exp(f\phi) - 1]}{\exp(f\phi) + 1}$$

$$F_2 = F_c + F_i \frac{2}{\exp(f\phi) + 1} \qquad (17\text{–}11)$$

Plot of Belt Tension vs. Initial Tension

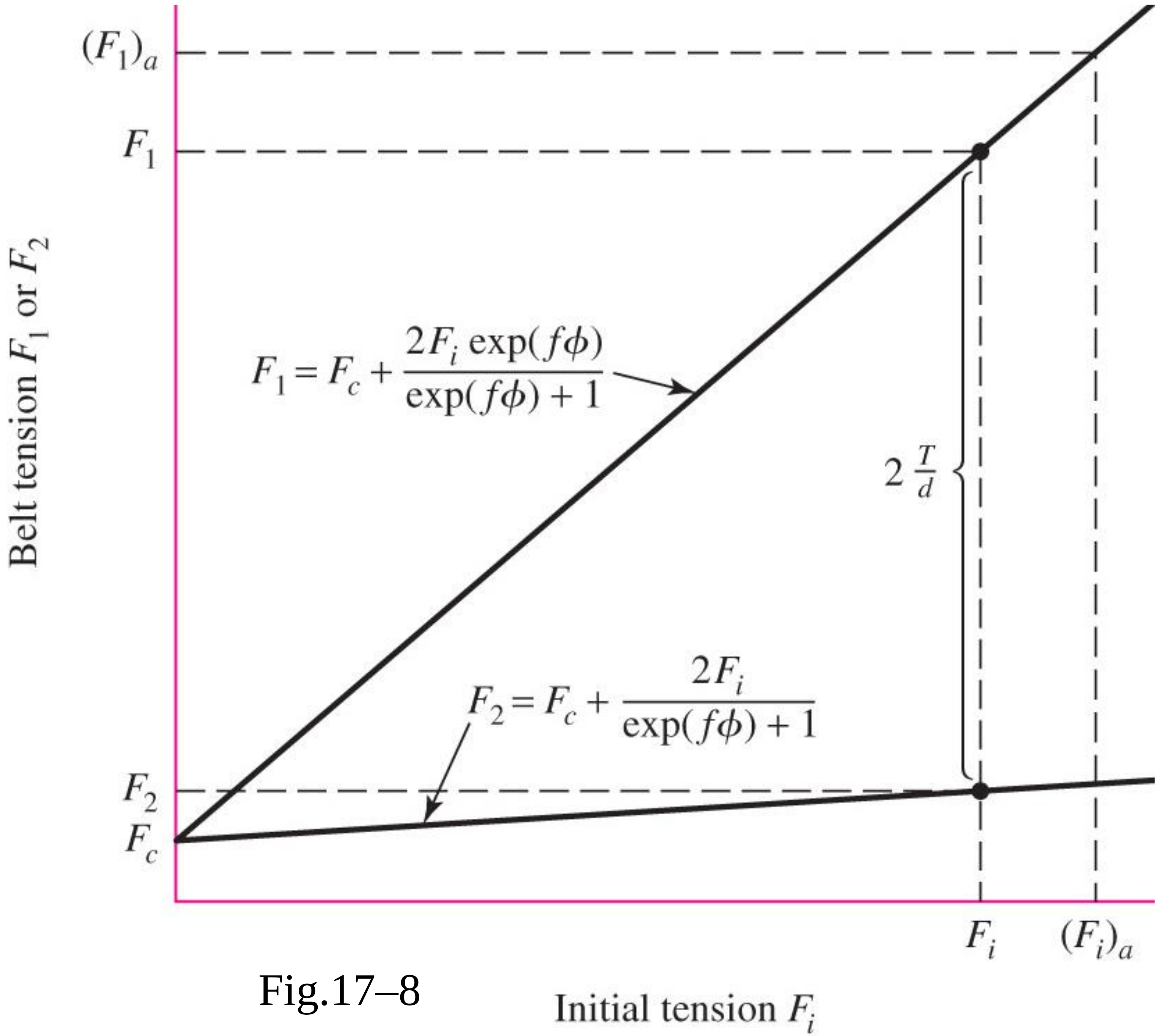


Fig.17–8

Transmitted Horsepower

$$H = \frac{(F_1 - F_2)V}{33\,000} \qquad (j)$$

Correction Factors

$$(F_1)_a = bF_aC_pC_v \qquad (17\text{–}12)$$

where $(F_1)_a$ = allowable largest tension, lbf

b = belt width, in

F_a = manufacturer's allowed tension, lbf/in

C_p = pulley correction factor (Table 17–4)

C_v = velocity correction factor

Velocity Correction Factor C_v for Leather Belts

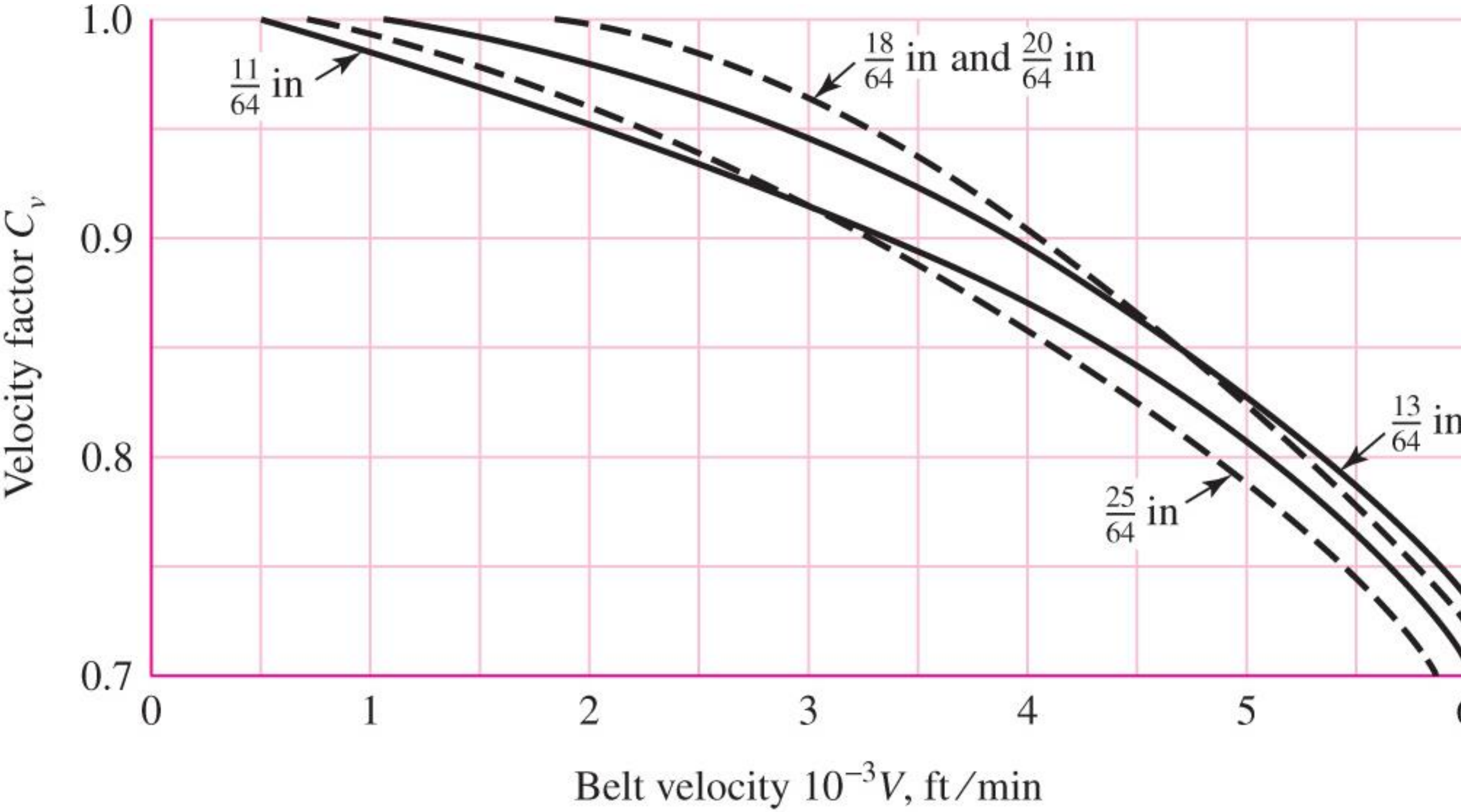


Fig.17–9

Pulley Correction Factor C_P for Flat Belts

Table 17–4

Pulley Correction Factor C_P for Flat Belts*

	Small-Pulley Diameter, in					
Material	**1.6 to 4**	**4.5 to 8**	**9 to 12.5**	**14, 16**	**18 to 31.5**	**Over 31.5**
Leather	0.5	0.6	0.7	0.8	0.9	1.0
Polyamide, F–0	0.95	1.0	1.0	1.0	1.0	1.0
F–1	0.70	0.92	0.95	1.0	1.0	1.0
F–2	0.73	0.86	0.96	1.0	1.0	1.0
A–2	0.73	0.86	0.96	1.0	1.0	1.0
A–3	—	0.70	0.87	0.94	0.96	1.0
A–4	—	—	0.71	0.80	0.85	0.92
A–5	—	—	—	0.72	0.77	0.91

*Average values of C_P for the given ranges were approximated from curves in the *Habasit Engineering Manual,* Habasit Belting, Inc., Chamblee (Atlanta), Ga.

Steps for Flat-Belt Analysis

1 Find $\exp(f\phi)$ from belt-drive geometry and friction
2 From belt geometry and speed find F_c
3 From $T = 63\,025 H_{\text{nom}} K_s n_d / n$ find necessary torque
4 From torque T find the necessary $(F_1)_a - F_2 = 2T/d$
5 From Tables 17–2 and 17–4, and Eq. (17–12) determine $(F_1)_a$.
6 Find F_2 from $(F_1)_a - [(F_1)_a - F_2]$
7 From Eq. (i) find the necessary initial tension F_i
8 Check the friction development, $f' < f$. Use Eq. (17–7) solved for f':

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c}$$

9 Find the factor of safety from $n_{fs} = H_a / (H_{\text{nom}} K_s)$

Properties of Some Flat- and Round-Belt Materials

Material	Specification	Size, in	Minimum Pulley Diameter, in	Allowable Tension per Unit Width at 600 ft/min, lbf/in	Specific Weight, lbf/in^3	Coefficient of Friction
Leather	1 ply	$t = \frac{11}{64}$	3	30	0.035–0.045	0.4
		$t = \frac{13}{64}$	$3\frac{1}{2}$	33	0.035–0.045	0.4
	2 ply	$t = \frac{18}{64}$	$4\frac{1}{2}$	41	0.035–0.045	0.4
		$t = \frac{20}{64}$	6[a]	50	0.035–0.045	0.4
		$t = \frac{23}{64}$	9[a]	60	0.035–0.045	0.4
Polyamide[b]	F–0[c]	$t = 0.03$	0.60	10	0.035	0.5
	F–1[c]	$t = 0.05$	1.0	35	0.035	0.5
	F–2[c]	$t = 0.07$	2.4	60	0.051	0.5
	A–2[c]	$t = 0.11$	2.4	60	0.037	0.8
	A–3[c]	$t = 0.13$	4.3	100	0.042	0.8
	A–4[c]	$t = 0.20$	9.5	175	0.039	0.8
	A–5[c]	$t = 0.25$	13.5	275	0.039	0.8
Urethane[d]	$w = 0.50$	$t = 0.062$	See	5.2[e]	0.038–0.045	0.7
	$w = 0.75$	$t = 0.078$	Table	9.8[e]	0.038–0.045	0.7
	$w = 1.25$	$t = 0.090$	17–3	18.9[e]	0.038–0.045	0.7
	Round	$d = \frac{1}{4}$	See	8.3[e]	0.038–0.045	0.7
		$d = \frac{3}{8}$	Table	18.6[e]	0.038–0.045	0.7
		$d = \frac{1}{2}$	17–3	33.0[e]	0.038–0.045	0.7
		$d = \frac{3}{4}$		74.3[e]	0.038–0.045	0.7

Table 17–2

Minimum Pulley Sizes for Flat and Round Urethane Belts

Belt Style	Belt Size, in	Ratio of Pulley Speed to Belt Length, rev/(ft · min) Up to 250	250 to 499	500 to 1000
Flat	0.50×0.062	0.38	0.44	0.50
	0.75×0.078	0.50	0.63	0.75
	1.25×0.090	0.50	0.63	0.75
Round	$\frac{1}{4}$	1.50	1.75	2.00
	$\frac{3}{8}$	2.25	2.62	3.00
	$\frac{1}{2}$	3.00	3.50	4.00
	$\frac{3}{4}$	5.00	6.00	7.00

Table 17–3

Crown Height and ISO Pulley Diameters for Flat Belts

ISO Pulley Diameter, in	Crown Height, in	ISO Pulley Diameter, in	Crown Height, in	
			$w \leq 10$ in	$w > 10$ in
1.6, 2, 2.5	0.012	12.5, 14	0.03	0.03
2.8, 3.15	0.012	12.5, 14	0.04	0.04
3.55, 4, 4.5	0.012	22.4, 25, 28	0.05	0.05
5, 5.6	0.016	31.5, 35.5	0.05	0.06
6.3, 7.1	0.020	40	0.05	0.06
8, 9	0.024	45, 50, 56	0.06	0.08
10, 11.2	0.030	63, 71, 80	0.07	0.10

*Crown should be rounded, not angled; maximum roughness is R_a = AA 63 μin.

Table 17–5

Example 17–1

A polyamide A-3 flat belt 6 in wide is used to transmit 15 hp under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 8 ft apart. The 6-in driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 18 in in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

(*a*) Estimate the centrifugal tension F_c and the torque T.

(*b*) Estimate the allowable F_1, F_2, F_i and allowable power H_a.

(*c*) Estimate the factor of safety. Is it satisfactory?

Belt 6 in $\times$ 0.130 in

15 hp

$$\gamma = 0.042 \ \frac{\text{lbf}}{\text{in}^3}$$

$d = 6$ in, $D = 18$ in

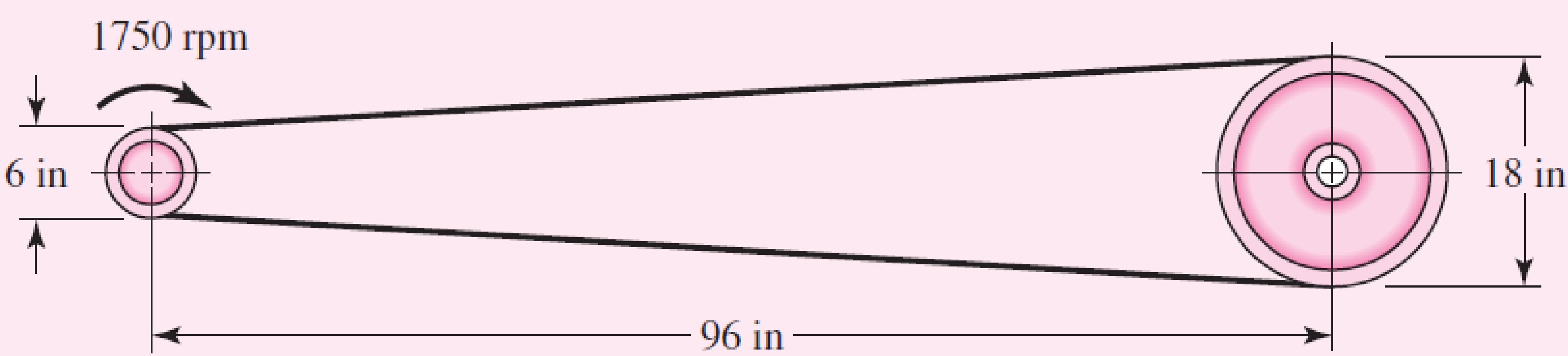


Fig.17–10

Example 17–1

(*a*) Eq. (17–1): $\phi = \theta_d = \pi - 2\sin^{-1}\left[\frac{18-6}{2(8)12}\right] = 3.0165 \text{ rad}$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(6)1750/12 = 2749 \text{ ft/min}$$

Table 17–2: $w = 12\gamma bt = 12(0.042)6(0.130) = 0.393 \text{ lbf/ft}$

Eq. (*e*): $F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.393}{32.17}\left(\frac{2749}{60}\right)^2 = 25.6 \text{ lbf}$

$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(15)1.25(1.1)}{1750}$$

$$= 742.8 \text{ lbf} \cdot \text{in}$$

Example 17–1

(b) The necessary $(F_1)_a - F_2$ to transmit the torque T, from Eq. (h), is

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$

From Table 17–2 $F_a = 100$ lbf. For polyamide belts $C_v = 1$, and from Table 17–4 $C_p = 0.70$. From Eq. (17–12) the allowable largest belt tension $(F_1)_a$ is

$$(F_1)_a = bF_aC_pC_v = 6(100)0.70(1) = 420 \text{ lbf}$$

then

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 420 - 247.6 = 172.4 \text{ lbf}$$

and from Eq. (i)

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{420 + 172.4}{2} - 25.6 = 270.6 \text{ lbf}$$

Example 17–1

The combination $(F_1)_a$, F_2, and F_i will transmit the design power of $15(1.25)(1.1) = 20.6$ hp and protect the belt. We check the friction development by solving Eq. (17–7) for f':

$$f' = \frac{1}{\phi} \ln \frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165} \ln \frac{420 - 25.6}{172.4 - 25.6} = 0.328$$

From Table 17–2, $f = 0.8$. Since $f' < f$, that is, $0.328 < 0.80$, there is no danger of slipping.

(c)

$$n_{fs} = \frac{H}{H_{\text{nom}} K_s} = \frac{20.6}{15(1.25)} = 1.1 \qquad \text{(as expected)}$$

The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 20.6 hp.

Belt-Tensioning Schemes

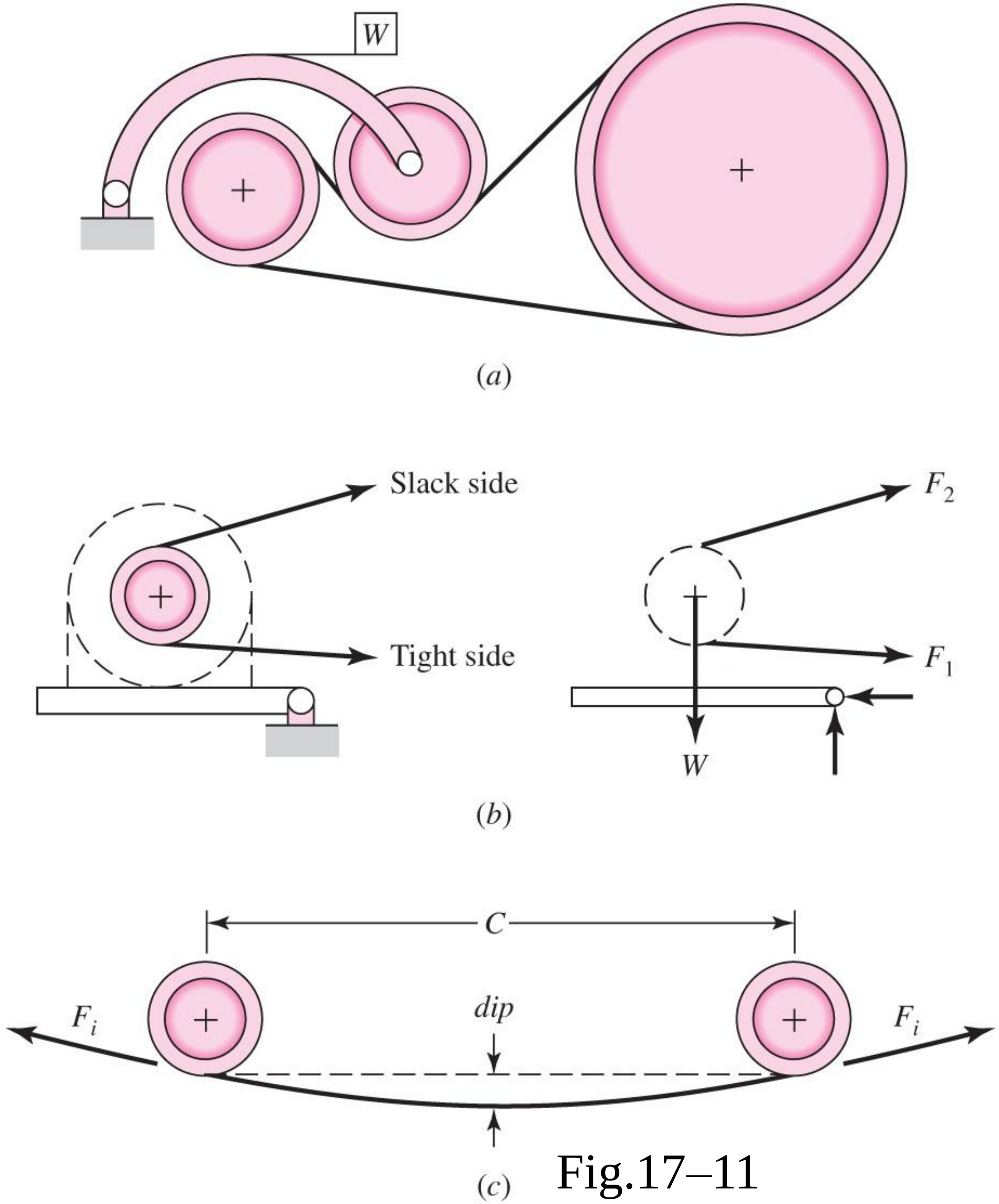


Fig.17–11

Relation of Dip to Initial Tension

$$dip = \frac{12(C/12)^2 w}{8F_i} = \frac{C^2 w}{96F_i} \qquad (17\text{–}13)$$

where dip = dip, in

C = center-to-center distance, in

w = weight per foot of the belt, lbf/ft

F_i = initial tension, lbf

Example 17–2

Design a flat-belt drive to connect horizontal shafts on 16-ft centers. The velocity ratio is to be 2.25:1. The angular speed of the small driving pulley is 860 rev/min, and the nominal power transmission is to be 60 hp under very light shock.

Solution

- Function: $H_{\text{nom}} = 60$ hp, 860 rev/min, 2.25:1 ratio, $K_s = 1.15$, $C = 16$ ft
- Design factor: $n_d = 1.05$
- Initial tension maintenance: catenary
- Belt material: polyamide
- Drive geometry, d, D
- Belt thickness: t
- Belt width: b

The last four could be design variables. Let's make a few more a priori decisions.

$d = 16$ in, $D = 2.25d = 2.25(16) = 36$ in.

Use polyamide A-3 belt; therefore $t = 0.13$ in and $C_v = 1$.

Example 17–2

Now there is one design decision remaining to be made, the belt width b.

Table 17–2: $\gamma = 0.042\ \text{lbf/in}^3 \qquad f = 0.8 \qquad F_a = 100\ \text{lbf/in at 600 rev/min}$

Table 17–4: $C_p = 0.94$

Eq. (17–12): $F_{1a} = b(100)0.94(1) = 94.0b\ \text{lbf}$ (1)

$$H_d = H_{\text{nom}} K_s n_d = 60(1.15)1.05 = 72.5\ \text{hp}$$

$$T = \frac{63\,025 H_d}{n} = \frac{63\,025(72.5)}{860} = 5310\ \text{lbf} \cdot \text{in}$$

Estimate $\exp(f\phi)$ for full friction development:

Eq. (17–1): $$\phi = \theta_d = \pi - 2\sin^{-1}\frac{36 - 16}{2(16)12} = 3.037\ \text{rad}$$

$$\exp(f\phi) = \exp[0.80(3.037)] = 11.35$$

Example 17–2

Estimate centrifugal tension F_c in terms of belt width b:

$$w = 12\gamma bt = 12(0.042)b(0.13) = 0.0655b \text{ lbf/ft}$$

$$V = \pi dn/12 = \pi(16)860/12 = 3602 \text{ ft/min}$$

Eq. (e):
$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.0655b}{32.17}\left(\frac{3602}{60}\right)^2 = 7.34b \text{ lbf} \qquad (2)$$

For design conditions, that is, at H_d power level, using Eq. (h) gives

$$(F_1)_a - F_2 = 2T/d = 2(5310)/16 = 664 \text{ lbf} \qquad (3)$$

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 94.0b - 664 \text{ lbf} \qquad (4)$$

Using Eq. (i) gives

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{94.0b + 94.0b - 664}{2} - 7.34b = 86.7b - 332 \text{ lbf} \qquad (5)$$

Example 17–2

Place friction development at its highest level, using Eq. (17–7):

$$f\phi = \ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \ln\frac{94.0b - 7.34b}{94.0b - 664 - 7.34b} = \ln\frac{86.7b}{86.7b - 664}$$

Solving the preceding equation for belt width b at which friction is fully developed gives

$$b = \frac{664}{86.7}\,\frac{\exp(f\phi)}{\exp(f\phi) - 1} = \frac{664}{86.7}\,\frac{11.38}{11.38 - 1} = 8.40 \text{ in}$$

A belt width greater than 8.40 in will develop friction less than $f = 0.80$. The manufacturer's data indicate that the next available larger width is 10 in.

Use 10-in-wide belt.
It follows that for a 10-in-wide belt

Eq. (2): $F_c = 7.34(10) = 73.4 \text{ lbf}$

Eq. (1): $(F_1)_a = 94(10) = 940 \text{ lbf}$

Eq. (4): $F_2 = 94(10) - 664 = 276 \text{ lbf}$

Eq. (5): $F_i = 86.7(10) - 332 = 535 \text{ lbf}$

Example 17–2

The transmitted power, from Eq. (3), is

$$H_t = \frac{[(F_1)_a - F_2]V}{33\,000} = \frac{664(3602)}{33\,000} = 72.5\text{ hp}$$

and the level of friction development f', from Eq. (17–7) is

$$f' = \frac{1}{\phi}\ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.037}\ln\frac{940 - 73.4}{276 - 73.4} = 0.479$$

which is less than $f = 0.8$, and thus is satisfactory. Had a 9-in belt width been available, the analysis would show $(F_1)_a = 846$ lbf, $F_2 = 182$ lbf, $F_i = 448$ lbf, and $f' = 0.63$. With a figure of merit available reflecting cost, thicker belts (A-4 or A-5) could be examined to ascertain which of the satisfactory alternatives is best. From Eq. (17–13) the catenary dip is

$$dip = \frac{C^2 w}{96F_i} = \frac{[16(12)]^2\,0.0655(10)}{96(535)} = 0.470\text{ in}$$

Variation of Flat-Belt Tensions at Some Cardinal Points

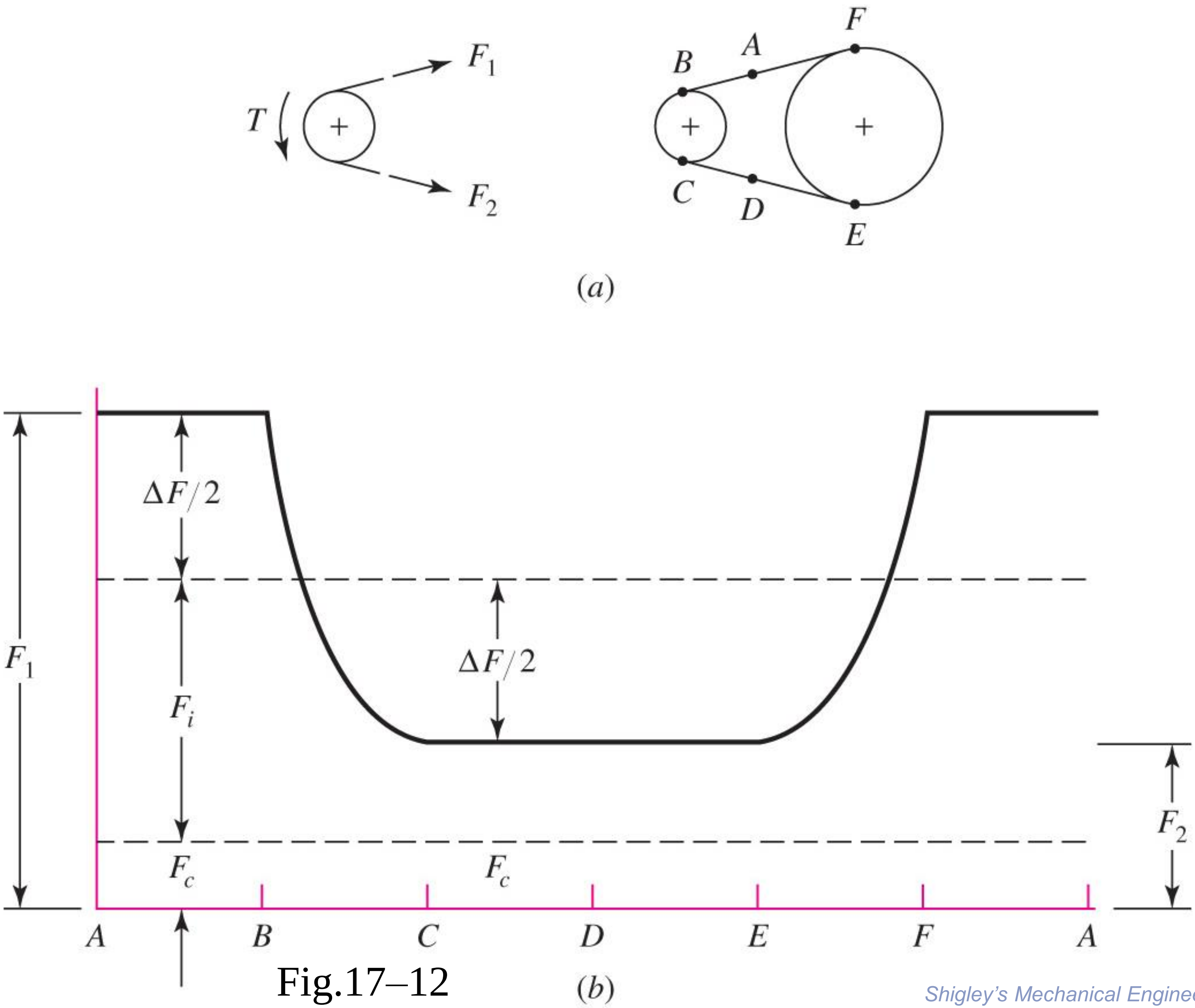


Fig.17–12

Flat Metal Belts

- Thin metal belts exhibit
  - High strength-to-weight ratio
  - Dimensional stability
  - Accurate timing
  - Usefulness to temperatures up to 700ºF
  - Good electrical and thermal conduction properties

Tensions and Torques in Thin Flat Metal Belt

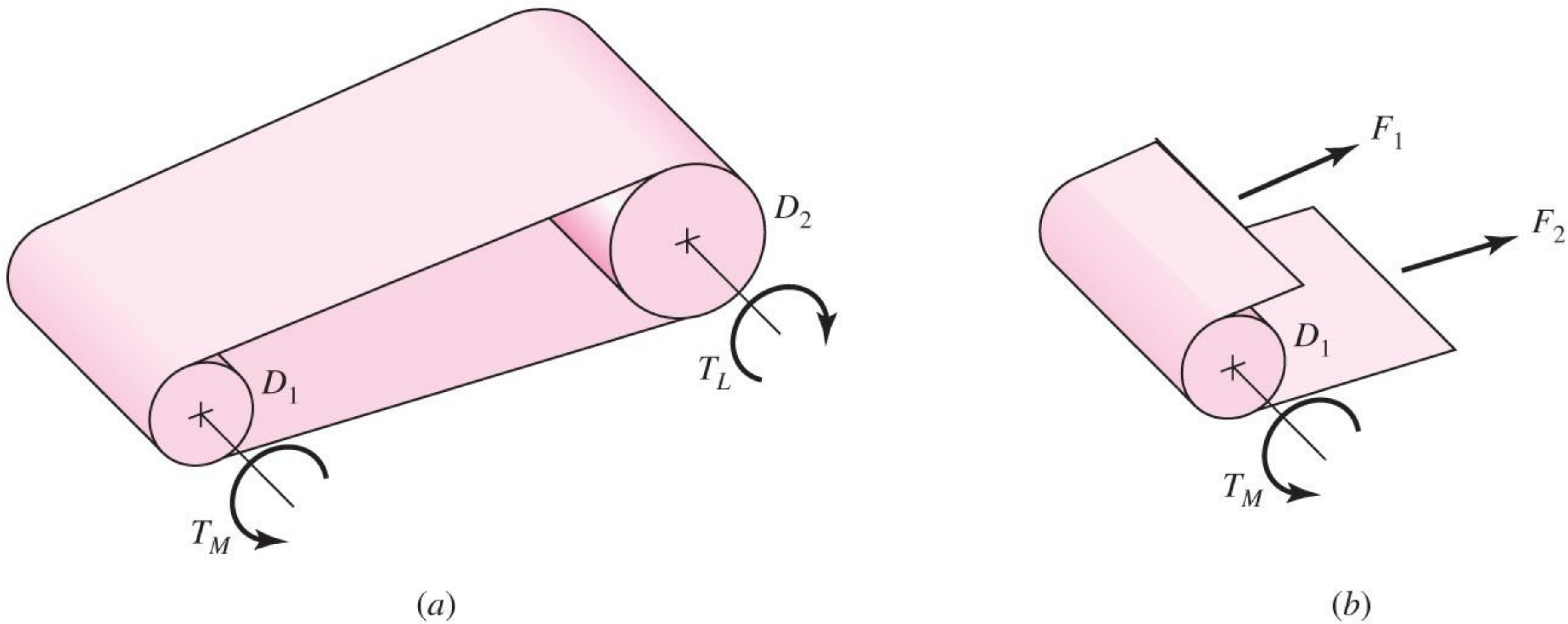


Fig.17–13

Bending Stress in Flat Metal Belt

$$\sigma_b = \frac{Et}{(1-\nu^2)D} = \frac{E}{(1-\nu^2)(D/t)} \qquad (17\text{–}14)$$

where E = Young's modulus

t = belt thickness

ν = Poisson's ratio

D = pulley diameter

Tensile Stresses in Flat Metal Belt

$$(\sigma)_1 = F_1/(bt)$$

$$(\sigma)_2 = F_2/(bt)$$

Largest tensile stress during a belt pass:

$$(\sigma_b)_1 + F_1/(bt)$$

Smallest tensile stress during a belt pass:

$$(\sigma_b)_2 + F_2/(bt)$$

Belt Life for Stainless Steel Friction Drives

$\frac{D}{t}$	Belt Passes
625	$\geq 10^6$
400	$0.500 \cdot 10^6$
333	$0.165 \cdot 10^6$
200	$0.085 \cdot 10^6$

Table 17–6

Regression Line for Stress and Passes

$$\sigma = 14\,169\,982 N^{-0.407} = 14.17(10^6) N_p^{-0.407} \qquad (17\text{–}15)$$

Minimum Pulley Diameter

Belt Thickness, in	Minimum Pulley Diameter, in
0.002	1.2
0.003	1.8
0.005	3.0
0.008	5.0
0.010	6.0
0.015	10
0.020	12.5
0.040	25.0

Table 17–7

Typical Material Properties for Metal Belts

Alloy	Yield Strength, kpsi	Young's Modulus, Mpsi	Poisson's Ratio
301 or 302 stainless steel	175	28	0.285
BeCu	170	17	0.220
1075 or 1095 carbon steel	230	30	0.287
Titanium	150	15	—
Inconel	160	30	0.284

Table 17–8

Steps for Selection of Metal Flat Belt

1 Find $\exp(f\phi)$ from geometry and friction

2 Find endurance strength

$$S_f = 14.17(10^6)N_p^{-0.407} \qquad \text{301, 302 stainless}$$

$$S_f = S_y/3 \qquad \text{others}$$

3 Allowable tension

$$F_{1a} = \left[S_f - \frac{Et}{(1-\nu^2)D}\right]tb = ab$$

4 $\Delta F = 2T/D$

5 $F_2 = F_{1a} - \Delta F = ab - \Delta F$

6 $F_i = \dfrac{F_{1a} + F_2}{2} = \dfrac{ab + ab - \Delta F}{2} = ab - \dfrac{\Delta F}{2}$

Steps for Selection of Metal Flat Belt

7 $$b_{\min} = \frac{\Delta F}{a} \frac{\exp(f\phi)}{\exp(f\phi) - 1}$$

8 Choose $b > b_{\min}$, $F_1 = ab$, $F_2 = ab - \Delta F$,

$$F_i = ab - \Delta F/2, \; T = \Delta F D/2$$

9 Check frictional development f':

$$f' = \frac{1}{\phi} \ln \frac{F_1}{F_2} \qquad f' < f$$

Example 17–3

A friction-drive stainless steel metal belt runs over two 4-in metal pulleys ($f = 0.35$). The belt thickness is to be 0.003 in. For a life exceeding 10^6 belt passes with smooth torque ($K_s = 1$), (*a*) select the belt if the torque is to be 30 lbf · in, and (*b*) find the initial tension F_i.

Solution

(*a*) From step 1, $\phi = \theta_d = \pi$, therefore $\exp(0.35\pi) = 3.00$. From step 2,

$$(S_f)_{10^6} = 14.17(10^6)(10^6)^{-0.407} = 51\ 210 \text{ psi}$$

From steps 3, 4, 5, and 6,

$$F_{1a} = \left[51\ 210 - \frac{28(10^6)0.003}{(1-0.285^2)4}\right]0.003b = 85.1b \text{ lbf} \qquad (1)$$

$$\Delta F = 2T/D = 2(30)/4 = 15 \text{ lbf}$$

$$F_2 = F_{1a} - \Delta F = 85.1b - 15 \text{ lbf} \qquad (2)$$

$$F_i = \frac{F_{1a} + F_2}{2} = \frac{85.1b + 15}{2} \text{ lbf} \qquad (3)$$

Example 17–3

From step 7,

$$b_{\min} = \frac{\Delta F}{a}\,\frac{\exp(f\phi)}{\exp(f\phi)-1} = \frac{15}{85.1}\,\frac{3.00}{3.00-1} = 0.264 \text{ in}$$

Select an available 0.75-in-wide belt 0.003 in thick.

Eq. (1): $F_1 = 85.1(0.75) = 63.8 \text{ lbf}$

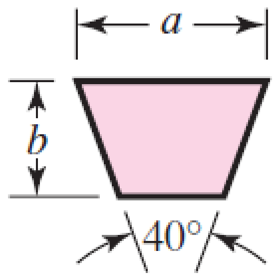
Eq. (2): $F_2 = 85.1(0.75) - 15 = 48.8 \text{ lbf}$

Eq. (3): $F_i = (63.8 + 48.8)/2 = 56.3 \text{ lbf}$

$$f' = \frac{1}{\phi}\ln\frac{F_1}{F_2} = \frac{1}{\pi}\ln\frac{63.8}{48.8} = 0.0853$$

Note $f' < f$, that is, $0.0853 < 0.35$.

Standard V-Belt Sections



Belt Section	Width *a*, in	Thickness *b*, in	Minimum Sheave Diameter, in	hp Range, One or More Belts
A	$\frac{1}{2}$	$\frac{11}{32}$	3.0	$\frac{1}{4}$–10
B	$\frac{21}{32}$	$\frac{7}{16}$	5.4	1–25
C	$\frac{7}{8}$	$\frac{17}{32}$	9.0	15–100
D	$1\frac{1}{4}$	$\frac{3}{4}$	13.0	50–250
E	$1\frac{1}{2}$	1	21.6	100 and up

Table 17–9

Inside Circumferences of Standard V-Belts

Section	Circumference, in
A	26, 31, 33, 35, 38, 42, 46, 48, 51, 53, 55, 57, 60, 62, 64, 66, 68, 71, 75, 78, 80, 85, 90, 96, 105, 112, 120, 128
B	35, 38, 42, 46, 48, 51, 53, 55, 57, 60, 62, 64, 65, 66, 68, 71, 75, 78, 79, 81, 83, 85, 90, 93, 97, 100, 103, 105, 112, 120, 128, 131, 136, 144, 158, 173, 180, 195, 210, 240, 270, 300
C	51, 60, 68, 75, 81, 85, 90, 96, 105, 112, 120, 128, 136, 144, 158, 162,173, 180, 195, 210, 240, 270, 300, 330, 360, 390, 420
D	120, 128, 144, 158, 162, 173, 180, 195, 210, 240, 270, 300, 330, 360,390, 420, 480, 540, 600, 660
E	180, 195, 210, 240, 270, 300, 330, 360, 390, 420, 480, 540, 600, 660

Table 17–10

Length Conversion Dimensions

Table 17–11

Length Conversion Dimensions (Add the listed quantity to the inside circumference to obtain the pitch length in inches).

Belt section	A	B	C	D	E
Quantity to be added	1.3	1.8	2.9	3.3	4.5

V-Belt Pitch Length and Center-to-Center Distance

$$L_p = 2C + \pi(D+d)/2 + (D-d)^2/(4C) \qquad (17\text{–}16a)$$

$$C = 0.25\left\{\left[L_p - \frac{\pi}{2}(D+d)\right] + \sqrt{\left[L_p - \frac{\pi}{2}(D+d)\right]^2 - 2(D-d)^2}\right\} \qquad (17\text{–}16b)$$

Horsepower Ratings of Standard V-Belts

Table 17–12

Belt Section	Sheave Pitch Diameter, in	Belt Speed, ft/min 1000	2000	3000	4000	5000
A	2.6	0.47	0.62	0.53	0.15	
	3.0	0.66	1.01	1.12	0.93	0.38
	3.4	0.81	1.31	1.57	1.53	1.12
	3.8	0.93	1.55	1.92	2.00	1.71
	4.2	1.03	1.74	2.20	2.38	2.19
	4.6	1.11	1.89	2.44	2.69	2.58
	5.0 and up	1.17	2.03	2.64	2.96	2.89
B	4.2	1.07	1.58	1.68	1.26	0.22
	4.6	1.27	1.99	2.29	2.08	1.24
	5.0	1.44	2.33	2.80	2.76	2.10
	5.4	1.59	2.62	3.24	3.34	2.82
	5.8	1.72	2.87	3.61	3.85	3.45
	6.2	1.82	3.09	3.94	4.28	4.00
	6.6	1.92	3.29	4.23	4.67	4.48
	7.0 and up	2.01	3.46	4.49	5.01	4.90
C	6.0	1.84	2.66	2.72	1.87	
	7.0	2.48	3.94	4.64	4.44	3.12
	8.0	2.96	4.90	6.09	6.36	5.52
	9.0	3.34	5.65	7.21	7.86	7.39
	10.0	3.64	6.25	8.11	9.06	8.89
	11.0	3.88	6.74	8.84	10.0	10.1
	12.0 and up	4.09	7.15	9.46	10.9	11.1
D	10.0	4.14	6.13	6.55	5.09	1.35
	11.0	5.00	7.83	9.11	8.50	5.62
	12.0	5.71	9.26	11.2	11.4	9.18
	13.0	6.31	10.5	13.0	13.8	12.2
	14.0	6.82	11.5	14.6	15.8	14.8
	15.0	7.27	12.4	15.9	17.6	17.0
	16.0	7.66	13.2	17.1	19.2	19.0
	17.0 and up	8.01	13.9	18.1	20.6	20.7
E	16.0	8.68	14.0	17.5	18.1	15.3
	18.0	9.92	16.7	21.2	23.0	21.5
	20.0	10.9	18.7	24.2	26.9	26.4
	22.0	11.7	20.3	26.6	30.2	30.5
	24.0	12.4	21.6	28.6	32.9	33.8
	26.0	13.0	22.8	30.3	35.1	36.7
	28.0 and up	13.4	23.7	31.8	37.1	39.1

Adjusted Power

$$H_a = K_1 K_2 H_{\text{tab}} \tag{17–17}$$

where H_a = allowable power, per belt

K_1 = angle-of-wrap correction factor, Table 17–13

K_2 = belt length correction factor, Table 17–14

Angle of Wrap Correction Factor

$\frac{D-d}{C}$	θ, deg	K_1 VV	K_1 V Flat
0.00	180	1.00	0.75
0.10	174.3	0.99	0.76
0.20	166.5	0.97	0.78
0.30	162.7	0.96	0.79
0.40	156.9	0.94	0.80
0.50	151.0	0.93	0.81
0.60	145.1	0.91	0.83
0.70	139.0	0.89	0.84
0.80	132.8	0.87	0.85
0.90	126.5	0.85	0.85
1.00	120.0	0.82	0.82
1.10	113.3	0.80	0.80
1.20	106.3	0.77	0.77
1.30	98.9	0.73	0.73
1.40	91.1	0.70	0.70
1.50	82.8	0.65	0.65

Table 17–13

Belt-Length Correction Factor

	Nominal Belt Length, in				
Length Factor	A Belts	B Belts	C Belts	D Belts	E Belts
0.85	Up to 35	Up to 46	Up to 75	Up to 128	
0.90	38–46	48–60	81–96	144–162	Up to 195
0.95	48–55	62–75	105–120	173–210	210–240
1.00	60–75	78–97	128–158	240	270–300
1.05	78–90	105–120	162–195	270–330	330–390
1.10	96–112	128–144	210–240	360–420	420–480
1.15	120 and up	158–180	270–300	480	540–600
1.20		195 and up	330 and up	540 and up	660

*Multiply the rated horsepower per belt by this factor to obtain the corrected horsepower.

Table 17–14

Belting Equation for V-Belt

$$\frac{F_1 - F_c}{F_2 - F_c} = \exp(0.5123\phi) \qquad (17\text{–}18)$$

Design Power for V-Belt

$$H_d = H_{\text{nom}} K_s n_d \tag{17–19}$$

where H_{nom} is the nominal power

K_s is the service factor given in Table 17–15

n_d is the design factor

Number of belts:

$$N_b \geq \frac{H_d}{H_a} \qquad N_b = 1, 2, 3, \ldots \tag{17–20}$$

Suggested Service Factors for V-Belt Drives

Driven Machinery	Source of Power: Normal Torque Characteristic	Source of Power: High or Nonuniform Torque
Uniform	1.0 to 1.2	1.1 to 1.3
Light shock	1.1 to 1.3	1.2 to 1.4
Medium shock	1.2 to 1.4	1.4 to 1.6
Heavy shock	1.3 to 1.5	1.5 to 1.8

Table 17–15

V-Belt Tensions

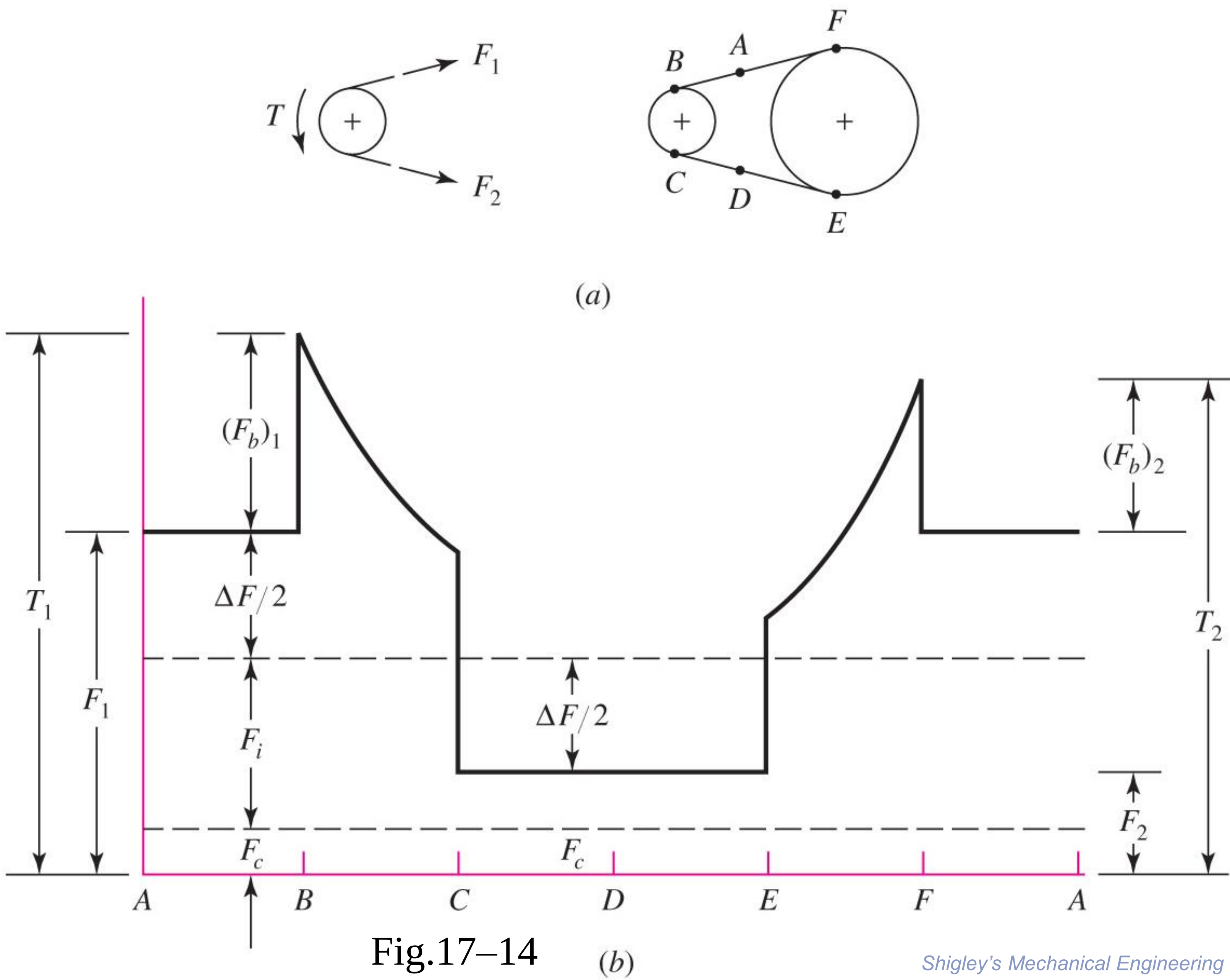


Fig.17–14

V-Belt Tensions

$$F_c = K_c \left(\frac{V}{1000} \right)^2 \qquad (17\text{–}21)$$

where K_c is from Table 17–16

$$\Delta F = \frac{63\,025 H_d / N_b}{n(d/2)} \qquad (17\text{–}22)$$

$$F_1 = F_c + \frac{\Delta F \exp(f\phi)}{\exp(f\phi) - 1} \qquad (17\text{–}23)$$

$$F_2 = F_1 - \Delta F \qquad (17\text{–}24)$$

$$F_i = \frac{F_1 + F_2}{2} - F_c \qquad (17\text{–}25)$$

Some V-Belt Parameters

Belt Section	K_b	K_c
A	220	0.561
B	576	0.965
C	1 600	1.716
D	5 680	3.498
E	10 850	5.041
3V	230	0.425
5V	1098	1.217
8V	4830	3.288

Table 17–16

V-Belt Factor of Safety

$$n_{fs} = \frac{H_a N_b}{H_{\text{nom}} K_s} \qquad (17\text{–}26)$$

V-Belt Tension vs. Passes

$$T_1 = F_1 + (F_b)_1 = F_1 + \frac{K_b}{d}$$

$$T_2 = F_1 + (F_b)_2 = F_1 + \frac{K_b}{D}$$

where K_b is given in Table 17–16

$$T^b N_P = K^b$$

$$N_P = \left[\left(\frac{K}{T_1} \right)^{-b} + \left(\frac{K}{T_2} \right)^{-b} \right]^{-1} \quad (17\text{–}27)$$

$$t = \frac{N_P L_p}{720V} \quad (17\text{–}28)$$

Durability Parameters for Some V-Belt Sections

Belt Section	10^8 to 10^9 Force Peaks K	10^8 to 10^9 Force Peaks b	10^9 to 10^{10} Force Peaks K	10^9 to 10^{10} Force Peaks b	Minimum Sheave Diameter, in
A	674	11.089			3.0
B	1193	10.926			5.0
C	2038	11.173			8.5
D	4208	11.105			13.0
E	6061	11.100			21.6
3V	728	12.464	1062	10.153	2.65
5V	1654	12.593	2394	10.283	7.1
8V	3638	12.629	5253	10.319	12.5

Table 17–17

Example 17–4

A 10-hp split-phase motor running at 1750 rev/min is used to drive a rotary pump, which operates 24 hours per day. An engineer has specified a 7.4-in small sheave, an 11-in large sheave, and three B112 belts. The service factor of 1.2 was augmented by 0.1 because of the continuous-duty requirement. Analyze the drive and estimate the belt life in passes and hours.

Solution

The peripheral speed V of the belt is

$$V = \pi\, dn/12 = \pi(7.4)1750/12 = 3390 \text{ ft/min}$$

Table 17–11: $L_p = L + L_c = 112 + 1.8 = 113.8$ in

Eq. (17–16*b*): $$C = 0.25\left\{\left[113.8 - \frac{\pi}{2}(11 + 7.4)\right] + \sqrt{\left[113.8 - \frac{\pi}{2}(11 + 7.4)\right]^2 - 2(11 - 7.4)^2}\right\}$$

$$= 42.4 \text{ in}$$

Example 17–4

Eq. (17–1): $$\phi = \theta_d = \pi - 2\sin^{-1}(11 - 7.4)/[2(42.4)] = 3.057 \text{ rad}$$

$$\exp[0.5123(3.057)] = 4.788$$

Interpolating in Table 17–12 for $V = 3390$ ft/min gives $H_{\text{tab}} = 4.693$ hp. The wrap angle in degrees is $3.057(180)/\pi = 175°$. From Table 17–13, $K_1 = 0.99$. From Table 17–14, $K_2 = 1.05$. Thus, from Eq. (17–17),

$$H_a = K_1 K_2 H_{\text{tab}} = 0.99(1.05)4.693 = 4.878 \text{ hp}$$

Eq. (17–19): $$H_d = H_{\text{nom}} K_s n_d = 10(1.2 + 0.1)(1) = 13 \text{ hp}$$

Eq. (17–20): $$N_b \geq H_d/H_a = 13/4.878 = 2.67 \rightarrow 3$$

From Table 17–16, $K_c = 0.965$. Thus, from Eq. (17–21),

$$F_c = 0.965(3390/1000)^2 = 11.1 \text{ lbf}$$

Eq. (17–22): $$\Delta F = \frac{63\,025(13)/3}{1750(7.4/2)} = 42.2 \text{ lbf}$$

Example 17–4

Eq. (17–23): $$F_1 = 11.1 + \frac{42.2(4.788)}{4.788 - 1} = 64.4 \text{ lbf}$$

Eq. (17–24): $$F_2 = F_1 - \Delta F = 64.4 - 42.2 = 22.2 \text{ lbf}$$

Eq. (17–25): $$F_i = \frac{64.4 + 22.2}{2} - 11.1 = 32.2 \text{ lbf}$$

Eq. (17–26): $$n_{fs} = \frac{H_a N_b}{H_{\text{nom}} K_s} = \frac{4.878(3)}{10(1.3)} = 1.13$$

Life: From Table 17–16, $K_b = 576$.

$$F_{b1} = \frac{K_b}{d} = \frac{576}{7.4} = 77.8 \text{ lbf}$$

$$F_{b2} = \frac{576}{11} = 52.4 \text{ lbf}$$

$$T_1 = F_1 + F_{b1} = 64.4 + 77.8 = 142.2 \text{ lbf}$$

$$T_2 = F_1 + F_{b2} = 64.4 + 52.4 = 116.8 \text{ lbf}$$

Example 17–4

From Table 17–17, $K = 1193$ and $b = 10.926$.

Eq. (17–27):
$$N_P = \left[\left(\frac{1193}{142.2}\right)^{-10.926} + \left(\frac{1193}{116.8}\right)^{-10.926}\right]^{-1} = 11(10^9)\text{ passes}$$

Since N_P is out of the validity range of Eq. (17–27), life is reported as greater than 10^9 passes. Then

Eq. (17–28):
$$t > \frac{10^9(113.8)}{720(3390)} = 46\,600\text{ h}$$

Timing Belts

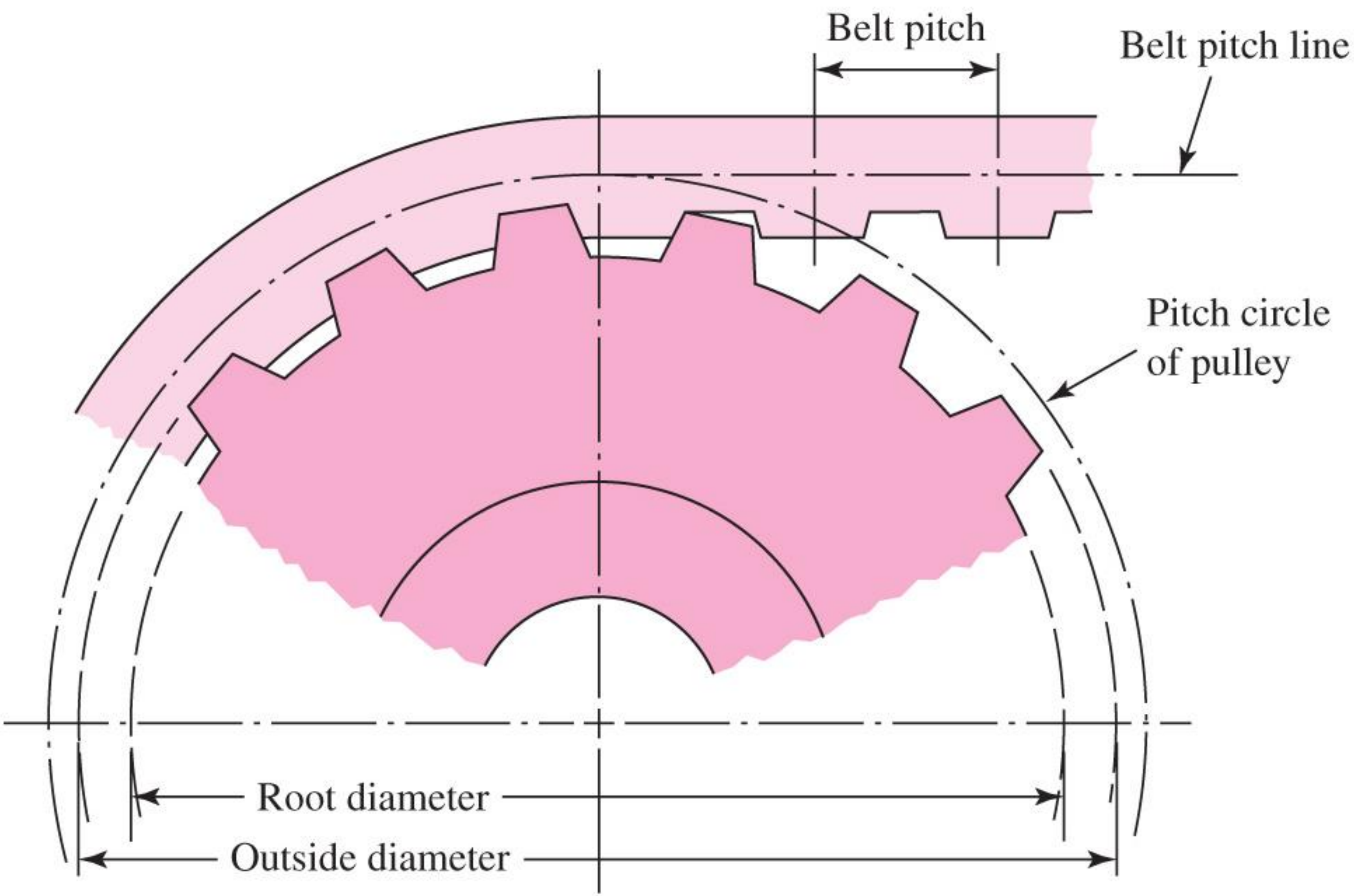


Fig.17–15

Standard Pitches of Timing Belts

Service	Designation	Pitch p, in
Extra light	XL	$\frac{1}{5}$
Light	L	$\frac{3}{8}$
Heavy	H	$\frac{1}{2}$
Extra heavy	XH	$\frac{7}{8}$
Double extra heavy	XXH	$1\frac{1}{4}$

Table 17–18

Roller Chain

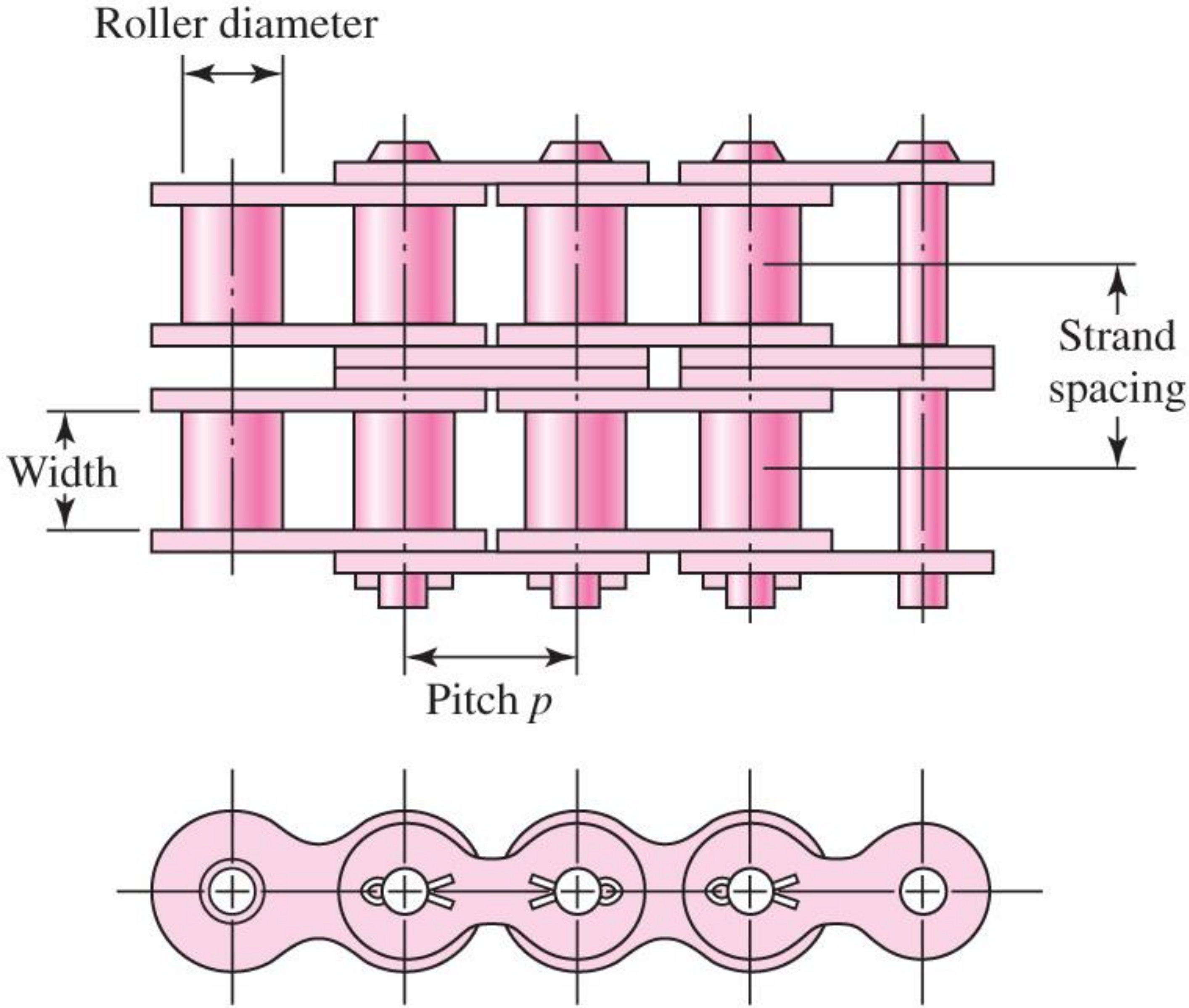


Fig.17–16

Dimensions of American Standard Roller Chains

ANSI Chain Number	Pitch, in (mm)	Width, in (mm)	Minimum Tensile Strength, lbf (N)	Average Weight, lbf/ft (N/m)	Roller Diameter, in (mm)	Multiple-Strand Spacing, in (mm)
25	0.250 (6.35)	0.125 (3.18)	780 (3 470)	0.09 (1.31)	0.130 (3.30)	0.252 (6.40)
35	0.375 (9.52)	0.188 (4.76)	1 760 (7 830)	0.21 (3.06)	0.200 (5.08)	0.399 (10.13)
41	0.500 (12.70)	0.25 (6.35)	1 500 (6 670)	0.25 (3.65)	0.306 (7.77)	— —
40	0.500 (12.70)	0.312 (7.94)	3 130 (13 920)	0.42 (6.13)	0.312 (7.92)	0.566 (14.38)
50	0.625 (15.88)	0.375 (9.52)	4 880 (21 700)	0.69 (10.1)	0.400 (10.16)	0.713 (18.11)
60	0.750 (19.05)	0.500 (12.7)	7 030 (31 300)	1.00 (14.6)	0.469 (11.91)	0.897 (22.78)
80	1.000 (25.40)	0.625 (15.88)	12 500 (55 600)	1.71 (25.0)	0.625 (15.87)	1.153 (29.29)
100	1.250 (31.75)	0.750 (19.05)	19 500 (86 700)	2.58 (37.7)	0.750 (19.05)	1.409 (35.76)
120	1.500 (38.10)	1.000 (25.40)	28 000 (124 500)	3.87 (56.5)	0.875 (22.22)	1.789 (45.44)
140	1.750 (44.45)	1.000 (25.40)	38 000 (169 000)	4.95 (72.2)	1.000 (25.40)	1.924 (48.87)
160	2.000 (50.80)	1.250 (31.75)	50 000 (222 000)	6.61 (96.5)	1.125 (28.57)	2.305 (58.55)
180	2.250 (57.15)	1.406 (35.71)	63 000 (280 000)	9.06 (132.2)	1.406 (35.71)	2.592 (65.84)
200	2.500 (63.50)	1.500 (38.10)	78 000 (347 000)	10.96 (159.9)	1.562 (39.67)	2.817 (71.55)
240	3.00 (76.70)	1.875 (47.63)	112 000 (498 000)	16.4 (239)	1.875 (47.62)	3.458 (87.83)

Table 17–19

Engagement of Chain and Sprocket

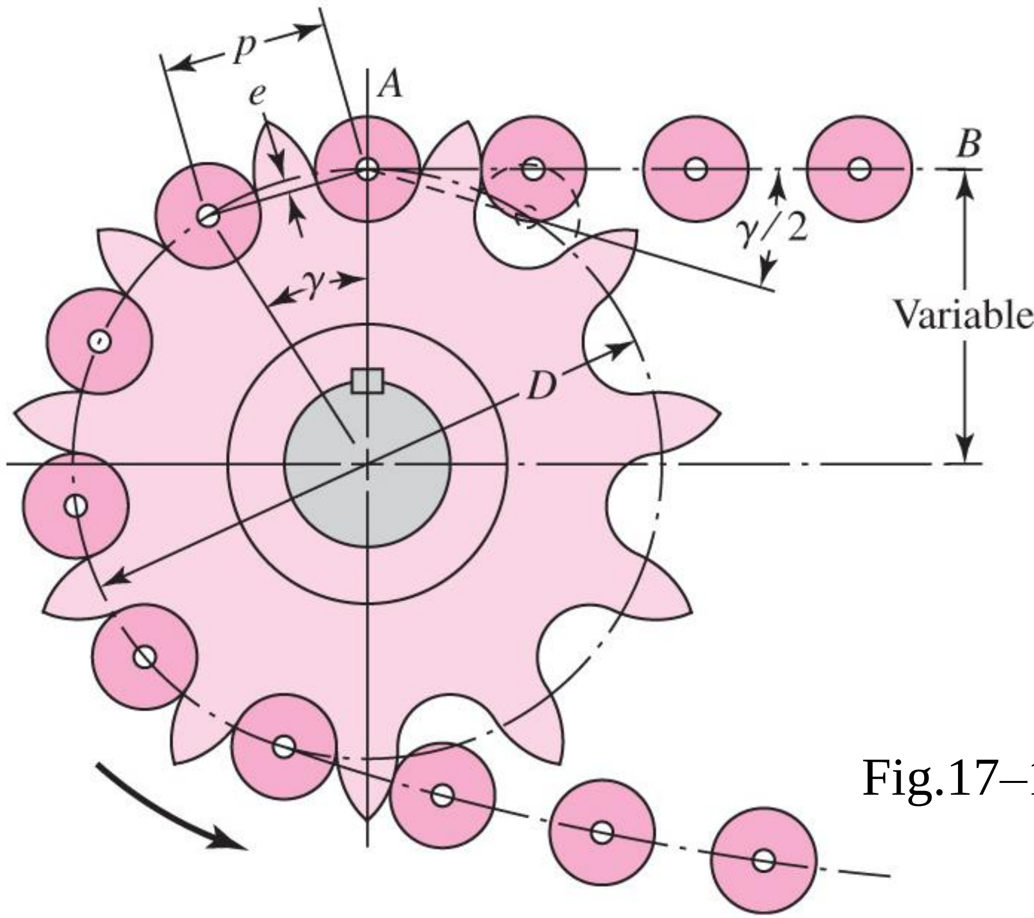


Fig.17–17

$$\sin\frac{\gamma}{2} = \frac{p/2}{D/2} \qquad \text{or} \qquad D = \frac{p}{\sin(\gamma/2)} \qquad (a)$$

$$D = \frac{p}{\sin(180°/N)} \qquad (17\text{–}29)$$

Chain Velocity

$$V = \frac{Npn}{12} \quad \text{feet per minute} \tag{17–30}$$

where N = number of sprocket teeth

p = chain pitch, in

n = sprocket speed, rev/min

$$v_{\max} = \frac{\pi D n}{12} = \frac{\pi n p}{12 \sin(\gamma/2)} \tag{b}$$

$$d = D \cos\frac{\gamma}{2} \tag{c}$$

$$v_{\min} = \frac{\pi d n}{12} = \frac{\pi n p}{12} \frac{\cos(\gamma/2)}{\sin(\gamma/2)} \tag{d}$$

Chordal Speed Variation

$$\frac{\Delta V}{V} = \frac{v_{\max} - v_{\min}}{V} = \frac{\pi}{N}\left[\frac{1}{\sin(180^\circ/N)} - \frac{1}{\tan(180^\circ/N)}\right] \qquad (17\text{–}31)$$

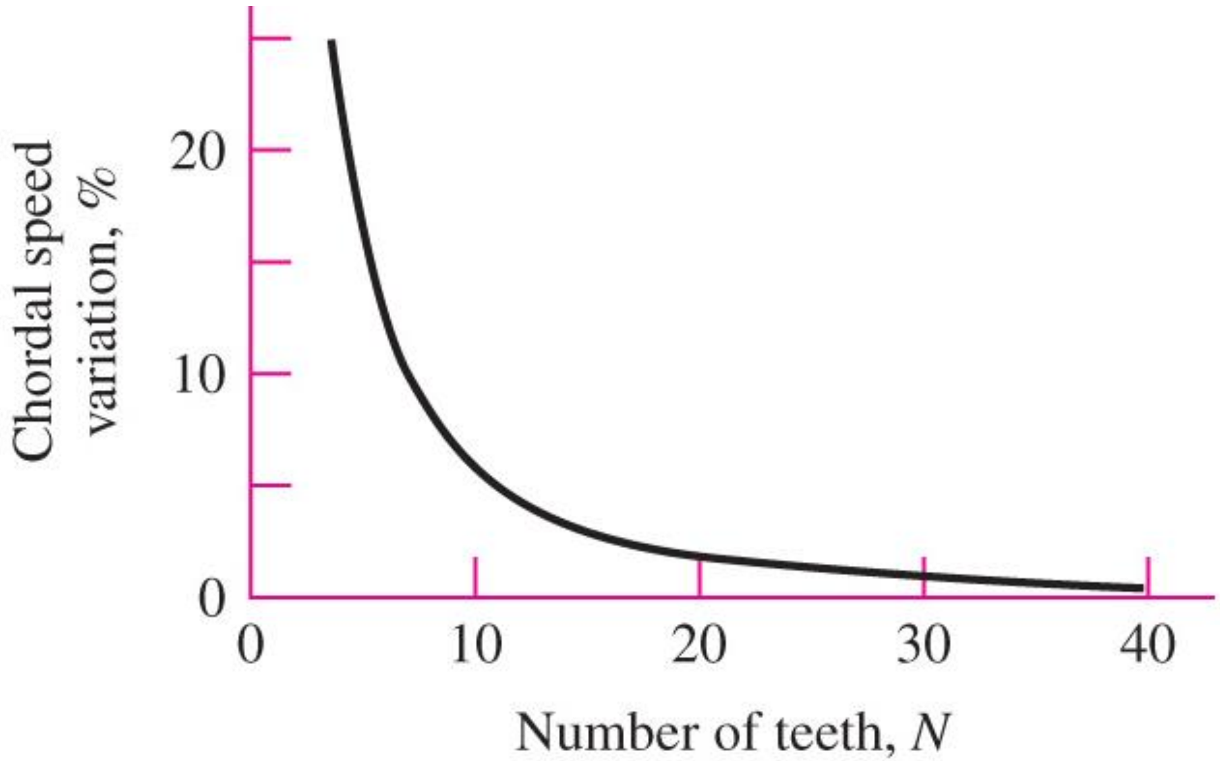


Fig.17–18

Roller Chain Rated Horsepower Capacity

Table 17–20

Rated Horsepower Capacity of Single-Strand Single-Pitch Roller Chain for a 17-Tooth Sprocket

Source: Compiled from ANSI B29.1-1975 information only section, and from B29.9-1958.

Sprocket Speed, rev/min	ANSI Chain Number 25	35	40	41	50	60
50	0.05	0.16	0.37	0.20	0.72	1.24
100	0.09	0.29	0.69	0.38	1.34	2.31
150	0.13*	0.41*	0.99*	0.55*	1.92*	3.32
200	0.16*	0.54*	1.29	0.71	2.50	4.30
300	0.23	0.78	1.85	1.02	3.61	6.20
400	0.30*	1.01*	2.40	1.32	4.67	8.03
500	0.37	1.24	2.93	1.61	5.71	9.81
600	0.44*	1.46*	3.45*	1.90*	6.72*	11.6
700	0.50	1.68	3.97	2.18	7.73	13.3
800	0.56*	1.89*	4.48*	2.46*	8.71*	15.0
900	0.62	2.10	4.98	2.74	9.69	16.7
1000	0.68*	2.31*	5.48	3.01	10.7	18.3
1200	0.81	2.73	6.45	3.29	12.6	21.6
1400	0.93*	3.13*	7.41	2.61	14.4	18.1
1600	1.05*	3.53*	8.36	2.14	12.8	14.8
1800	1.16	3.93	8.96	1.79	10.7	12.4
2000	1.27*	4.32*	7.72*	1.52*	9.23*	10.6
2500	1.56	5.28	5.51*	1.10*	6.58*	7.57
3000	1.84	5.64	4.17	0.83	4.98	5.76

Type A — Type B — Type C

Roller Chain Rated Horsepower Capacity

Table 17–20

Rated Horsepower Capacity of Single-Strand Single-Pitch Roller Chain for a 17-Tooth Sprocket (*Continued*)

Sprocket Speed, rev/min		ANSI Chain Number 80	100	120	140	160	180	200	240
50	Type A	2.88	5.52	9.33	14.4	20.9	28.9	38.4	61.8
100		5.38	10.3	17.4	26.9	39.1	54.0	71.6	115
150		7.75	14.8	25.1	38.8	56.3	77.7	103	166
200		10.0	19.2	32.5	50.3	72.9	101	134	215
300		14.5	27.7	46.8	72.4	105	145	193	310
400		18.7	35.9	60.6	93.8	136	188	249	359
500	Type B	22.9	43.9	74.1	115	166	204	222	0
600		27.0	51.7	87.3	127	141	155	169	
700		31.0	59.4	89.0	101	112	123	0	
800		35.0	63.0	72.8	82.4	91.7	101		
900		39.9	52.8	61.0	69.1	76.8	84.4		
1000		37.7	45.0	52.1	59.0	65.6	72.1		
1200		28.7	34.3	39.6	44.9	49.9	0		
1400		22.7	27.2	31.5	35.6	0			
1600		18.6	22.3	25.8	0				
1800		15.6	18.7	21.6					
2000		13.3	15.9	0					
2500		9.56	0.40						
3000		7.25	0						
Type C					Type C′				

Available Sprocket Tooth Counts

Table 17–21

Single-Strand Sprocket Tooth Counts Available from One Supplier*

No.	Available Sprocket Tooth Counts
25	8-30, 32, 34, 35, 36, 40, 42, 45, 48, 54, 60, 64, 65, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
35	4-45, 48, 52, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
41	6-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
40	8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
50	8-60, 64, 65, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
60	8-60, 62, 63, 64, 65, 66, 67, 68, 70, 72, 76, 80, 84, 90, 95, 96, 102, 112, 120
80	8-60, 64, 65, 68, 70, 72, 76, 78, 80, 84, 90, 95, 96, 102, 112, 120
100	8-60, 64, 65, 67, 68, 70, 72, 74, 76, 80, 84, 90, 95, 96, 102, 112, 120
120	9-45, 46, 48, 50, 52, 54, 55, 57, 60, 64, 65, 67, 68, 70, 72, 76, 80, 84, 90, 96, 102, 112, 120
140	9-28, 30, 31, 32, 33, 34, 35, 36, 37, 39, 40, 42, 43, 45, 48, 54, 60, 64, 65, 68, 70, 72, 76, 80, 84, 96
160	8-30, 32–36, 38, 40, 45, 46, 50, 52, 53, 54, 56, 57, 60, 62, 63, 64, 65, 66, 68, 70, 72, 73, 80, 84, 96
180	13-25, 28, 35, 39, 40, 45, 54, 60
200	9-30, 32, 33, 35, 36, 39, 40, 42, 44, 45, 48, 50, 51, 54, 56, 58, 59, 60, 63, 64, 65, 68, 70, 72
240	9-30, 32, 35, 36, 40, 44, 45, 48, 52, 54, 60

Tooth Correction Factors K_1

Number of Teeth on Driving Sprocket	K_1 Pre-extreme Horsepower	K_1 Post-extreme Horsepower
11	0.62	0.52
12	0.69	0.59
13	0.75	0.67
14	0.81	0.75
15	0.87	0.83
16	0.94	0.91
17	1.00	1.00
18	1.06	1.09
19	1.13	1.18
20	1.19	1.28
N	$(N_1/17)^{1.08}$	$(N_1/17)^{1.5}$

Table 17–22

Multiple-Strand Factors K_2

Number of Strands	K_2
1	1.0
2	1.7
3	2.5
4	3.3
5	3.9
6	4.6
8	6.0

Table 17–23

Nominal Power Ratings for Chain

- From American Chain Association publication *Chains for Power Transmission and Materials Handling*
- For single-strand chain
- Nominal power, link-plate limited

$$H_1 = 0.004 N_1^{1.08} n_1^{0.9} p^{(3-0.07p)} \qquad \text{hp} \qquad (17\text{–}32)$$

- Nominal power, roller-limited

$$H_2 = \frac{1000 K_r N_1^{1.5} p^{0.8}}{n_1^{1.5}} \qquad \text{hp} \qquad (17\text{–}33)$$

where N_1 = number of teeth in the smaller sprocket

n_1 = sprocket speed, rev/min

p = pitch of the chain, in

K_r = 29 for chain numbers 25, 35; 3.4 for chain 41; and 17 for chains 40–240

Chain Dimensions

- Chain length in pitches

$$\frac{L}{p} \doteq \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p} \qquad (17\text{–}34)$$

- Center-to-center distance

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\,\right] \qquad (17\text{–}35)$$

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} \qquad (17\text{–}36)$$

Chain Drive Power

- Allowable power

$$H_a = K_1 K_2 H_{\text{tab}} \qquad (17\text{–}37)$$

where K_1 = correction factor for tooth number other than 17 (Table 17–22)

K_2 = strand correction (Table 17–23)

- Power that must be transmitted

$$H_d = H_{\text{nom}} K_s n_d \qquad (17\text{–}38)$$

Variations in Tabulated Power Conditions

- Power ratings in Table 17–20 are for chains of 100 pitch length and 17-tooth sprocket.
- For deviations from this,

$$H_2 = 1000\left[K_r\left(\frac{N_1}{n_1}\right)^{1.5} p^{0.8}\left(\frac{L_p}{100}\right)^{0.4}\left(\frac{15\,000}{h}\right)^{0.4}\right] \qquad (17\text{–}39)$$

- From a deviation viewpoint,

$$\frac{H_2^{2.5}h}{N_1^{3.75}L_p} = \text{constant} \qquad (17\text{–}40)$$

Recommended Maximum Chain Speed

$$n_1 \le 1000 \left[\frac{82.5}{7.95^p (1.0278)^{N_1} (1.323)^{F/1000}} \right]^{1/(1.59 \log p + 1.873)} \qquad \text{rev/min}$$

where F is the chain tension in pounds

Example 17–5

Select drive components for a 2:1 reduction, 90-hp input at 300 rev/min, moderate shock, an abnormally long 18-hour day, poor lubrication, cold temperatures, dirty surroundings, short drive $C/p = 25$.

Solution

Function: $H_{\text{nom}} = 90$ hp, $n_1 = 300$ rev/min, $C/p = 25$, $K_s = 1.3$
Design factor: $n_d = 1.5$
Sprocket teeth: $N_1 = 17$ teeth, $N_2 = 34$ teeth, $K_1 = 1$, $K_2 = 1, 1.7, 2.5, 3.3$
Chain number of strands:

$$H_{\text{tab}} = \frac{n_d K_s H_{\text{nom}}}{K_1 K_2} = \frac{1.5(1.3)90}{(1)K_2} = \frac{176}{K_2}$$

Example 17–5

Form a table:

Number of Strands	176/K2 (Table 17–23)	Chain Number (Table 17–19)	Lubrication Type
1	$176/1 = 176$	200	C′
2	$176/1.7 = 104$	160	C
3	$176/2.5 = 70.4$	140	B
4	$176/3.3 = 53.3$	140	B

3 strands of number 140 chain (H_{tab} is 72.4 hp).

Number of pitches in the chain:

$$\frac{L}{p} = \frac{2C}{p} + \frac{N_1 + N_2}{2} + \frac{(N_2 - N_1)^2}{4\pi^2 C/p}$$

$$= 2(25) + \frac{17 + 34}{2} + \frac{(34 - 17)^2}{4\pi^2(25)} = 75.79 \text{ pitches}$$

Use 76 pitches. Then $L/p = 76$.

Example 17–5

Identify the center-to-center distance: From Eqs. (17–35) and (17–36),

$$A = \frac{N_1 + N_2}{2} - \frac{L}{p} = \frac{17 + 34}{2} - 76 = -50.5$$

$$C = \frac{p}{4}\left[-A + \sqrt{A^2 - 8\left(\frac{N_2 - N_1}{2\pi}\right)^2}\right]$$

$$= \frac{p}{4}\left[50.5 + \sqrt{50.5^2 - 8\left(\frac{34 - 17}{2\pi}\right)^2}\right] = 25.104p$$

For a 140 chain, $p = 1.75$ in. Thus,

$$C = 25.104p = 25.104(1.75) = 43.93 \text{ in}$$

Lubrication: Type B
Comment: This is operating on the pre-extreme portion of the power, so durability estimates other than 15 000 h are not available. Given the poor operating conditions, life will be much shorter.

Types of Wire Rope

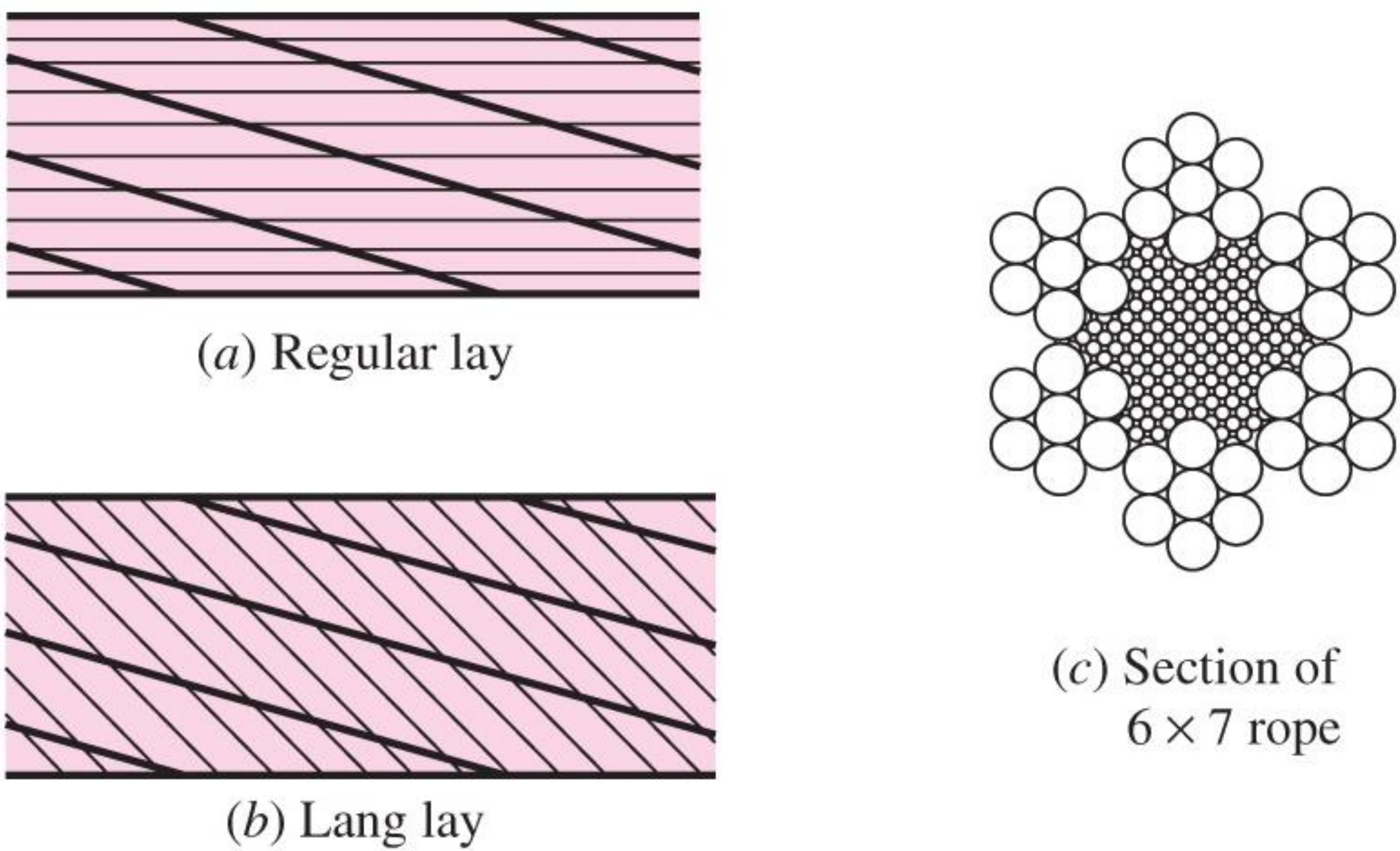


Fig.17–19

Stress in Wire Rope

$$M = \frac{EI}{\rho} \qquad \text{and} \qquad M = \frac{\sigma I}{c} \qquad (a)$$

$$\sigma = \frac{Ec}{\rho} \qquad (b)$$

$$c = d_w/2$$

where d_w is the wire diameter

$$\sigma = E_r \frac{d_w}{D} \qquad (c)$$

where E_r is the *modulus of elasticity of the rope*, not the wire

Wire-Rope Data

Rope	Weight per Foot, lbf	Minimum Sheave Diameter, in	Standard Sizes d, in	Material	Size of Outer Wires	Modulus of Elasticity,* Mpsi	Strength,† kpsi
6 × 7 haulage	$1.50d^2$	$42d$	$\frac{1}{4}$–$1\frac{1}{2}$	Monitor steel	$d/9$	14	100
				Plow steel	$d/9$	14	88
				Mild plow steel	$d/9$	14	76
6 × 19 standard hoisting	$1.60d^2$	$26d$–$34d$	$\frac{1}{4}$–$2\frac{3}{4}$	Monitor steel	$d/13$–$d/16$	12	106
				Plow steel	$d/13$–$d/16$	12	93
				Mild plow steel	$d/13$–$d/16$	12	80
6 × 37 special flexible	$1.55d^2$	$18d$	$\frac{1}{4}$–$3\frac{1}{2}$	Monitor steel	$d/22$	11	100
				Plow steel	$d/22$	11	88
8 × 19 extra flexible	$1.45d^2$	$21d$–$26d$	$\frac{1}{4}$–$1\frac{1}{2}$	Monitor steel	$d/15$–$d/19$	10	92
				Plow steel	$d/15$–$d/19$	10	80
7 × 7 aircraft	$1.70d^2$	—	$\frac{1}{16}$–$\frac{3}{8}$	Corrosion-resistant steel	—	—	124
				Carbon steel	—	—	124
7 × 9 aircraft	$1.75d^2$	—	$\frac{1}{8}$–$1\frac{3}{8}$	Corrosion-resistant steel	—	—	135
				Carbon steel	—	—	143
19-wire aircraft	$2.15d^2$	—	$\frac{1}{32}$–$\frac{5}{16}$	Corrosion-resistant steel	—	—	165
				Carbon steel	—	—	165

Table 17–24

Equivalent Bending Load

- Wire rope tension giving same tensile stress as sheave bending is called *equivalent bending load* F_b

$$F_b = \sigma A_m = \frac{E_r d_w A_m}{D} \qquad (17\text{–}41)$$

Percent Strength Loss

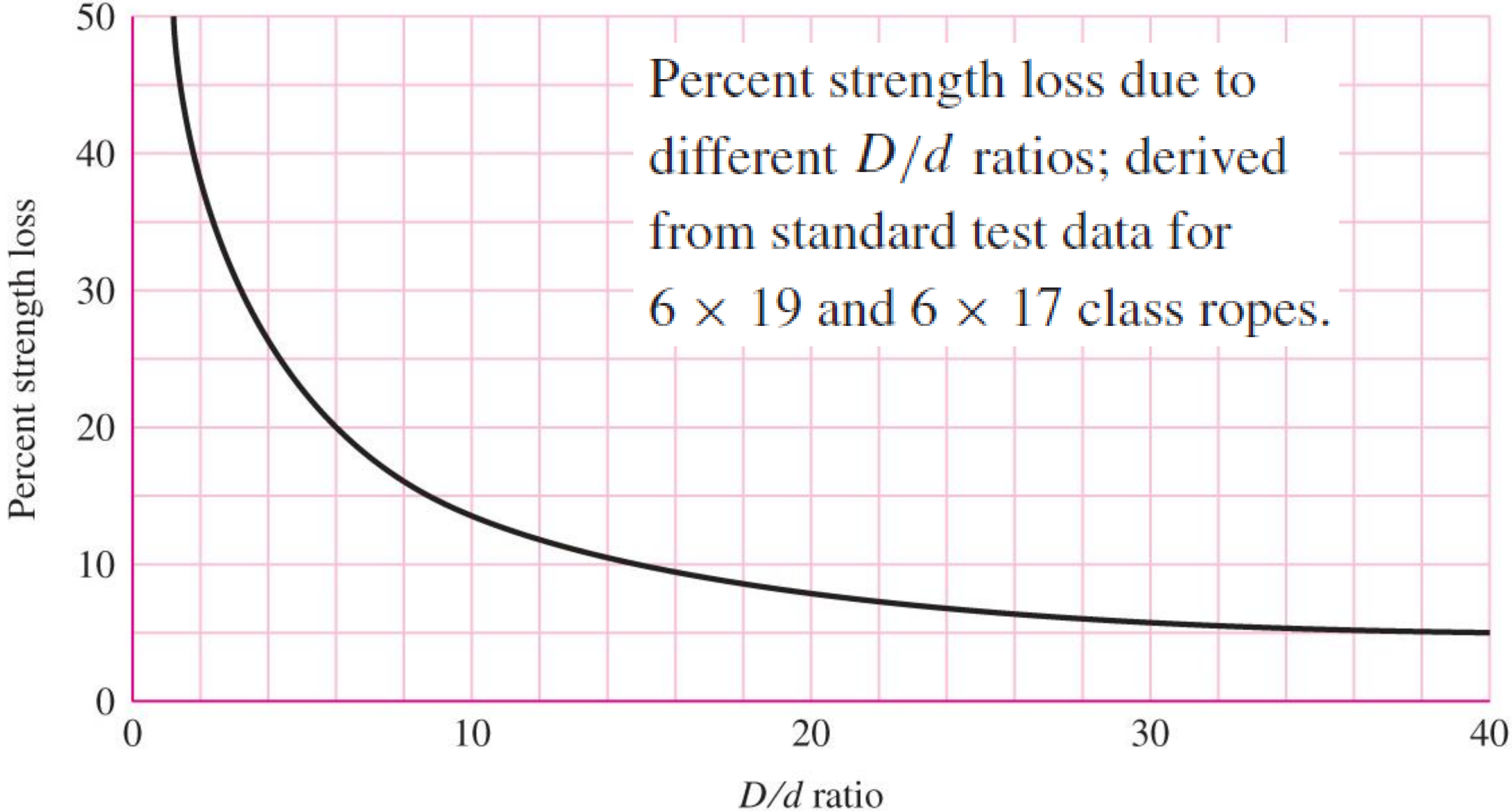


Fig.17–20

Minimum Factors of Safety for Wire Rope

Track cables	3.2	Passenger elevators, ft/min:	
Guys	3.5	50	7.60
Mine shafts, ft:		300	9.20
Up to 500	8.0	800	11.25
1000–2000	7.0	1200	11.80
2000–3000	6.0	1500	11.90
Over 3000	5.0	Freight elevators, ft/min:	
Hoisting	5.0	50	6.65
Haulage	6.0	300	8.20
Cranes and derricks	6.0	800	10.00
Electric hoists	7.0	1200	10.50
Hand elevators	5.0	1500	10.55
Private elevators	7.5	Powered dumbwaiters, ft/min:	
Hand dumbwaiter	4.5	50	4.8
Grain elevators	7.5	300	6.6
		500	8.0

Table 17–25

Bearing Pressure of Wire Rope in Sheave Groove

$$p = \frac{2F}{dD} \qquad (17\text{–}42)$$

where F = tensile force on rope

d = rope diameter

D = sheave diameter

Maximum Allowable Bearing Pressures (in psi)

	Sheave Material				
Rope	Wood[a]	Cast Iron[b]	Cast Steel[c]	Chilled Cast Irons[d]	Manganese Steel[e]
Regular lay:					
6 × 7	150	300	550	650	1470
6 × 19	250	480	900	1100	2400
6 × 37	300	585	1075	1325	3000
8 × 19	350	680	1260	1550	3500
Lang lay:					
6 × 7	165	350	600	715	1650
6 × 19	275	550	1000	1210	2750
6 × 37	330	660	1180	1450	3300

[a]On end grain of beech, hickory, or gum.

[b]For H_B (min.) = 125.

[c]30–40 carbon; H_B (min.) = 160.

[d]Use only with uniform surface hardness.

[e]For high speeds with balanced sheaves having ground surfaces.

Table 17–26

Relation Between Fatigue Life of Wire Rope and Sheave Pressure

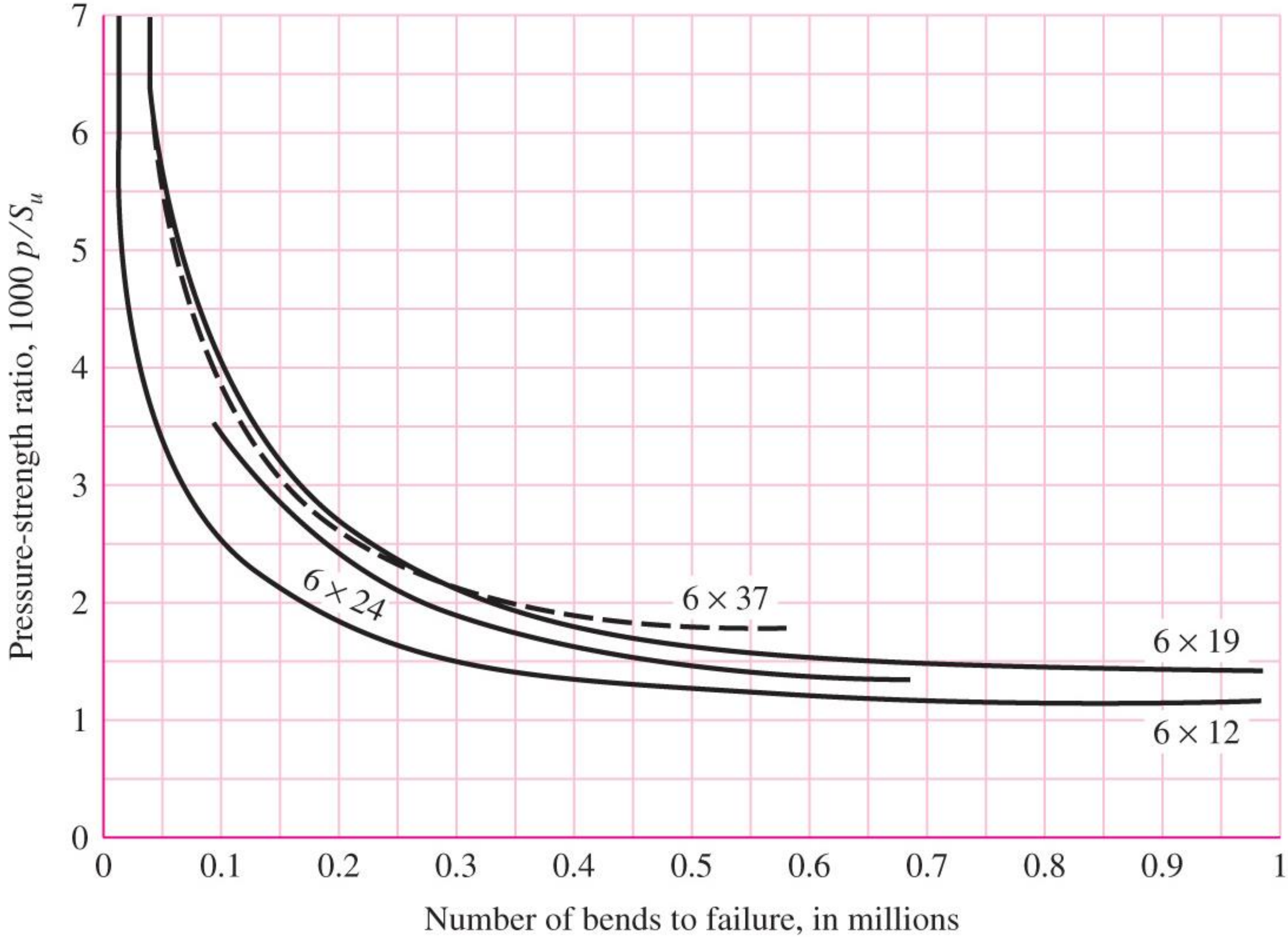


Fig.17–21

Fatigue of Wire Rope

- Fig. 17–21does not preclude failure by fatigue or wear
- It does show long life if p/S_u is less than 0.001.
- Substituting this ratio in Eq. (17–42),

$$S_u = \frac{2000F}{dD} \tag{17–43}$$

- Dividing both sides of Eq. (17–42) by S_u and solving for F, gives allowable fatigue tension,

$$F_f = \frac{(p/S_u)S_u d D}{2} \tag{17–44}$$

- Factor of safety for fatigue is

$$n_f = \frac{F_f - F_b}{F_t} \tag{17–45}$$

Typical Strength of Individual Wires

Improved plow steel (monitor)	$240 < S_u < 280$ kpsi
Plow steel	$210 < S_u < 240$ kpsi
Mild plow steel	$180 < S_u < 210$ kpsi

Service-Life Curve Based on Bending and Tensile Stresses

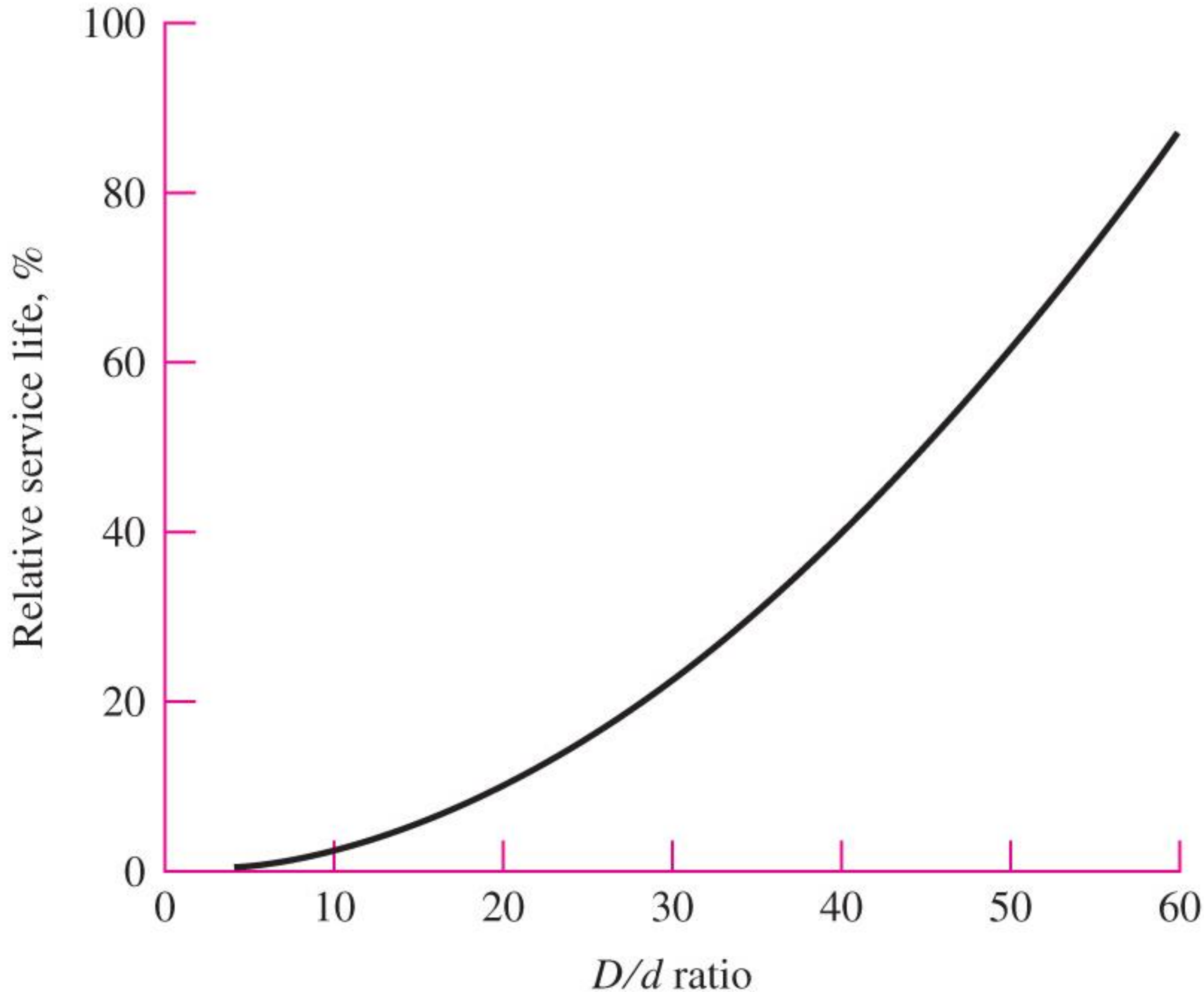


Fig.17–22

Some Wire-Rope Properties

Table 17–27

Some Useful Properties of 6 × 7, 6 × 19, and 6 × 37 Wire Ropes

Wire Rope	Weight per Foot w, lbf/ft	Weight per Foot Including Core w, lbf/ft	Minimum Sheave Diameter D, in	Better Sheave Diameter D, in	Diameter of Wires d_w, in	Area of Metal A_m, in^2	Rope Young's Modulus E_r, psi
6 × 7	$1.50d^2$		$42d$	$72d$	$0.111d$	$0.38d^2$	13×10^6
6 × 19	$1.60d^2$	$1.76d^2$	$30d$	$45d$	$0.067d$	$0.40d^2$	12×10^6
6 × 37	$1.55d^2$	$1.71d^2$	$18d$	$27d$	$0.048d$	$0.40d^2$	12×10^6

Working Equations for Mine-Hoist Problem

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) \qquad (17\text{–}46)$$

where W = weight at the end of the rope (cage and load), lbf

m = number of wire ropes supporting the load

w = weight/foot of the wire rope, lbf/ft

l = suspended length of rope, ft

a = maximum acceleration/deceleration experienced, ft/s^2

g = acceleration of gravity, ft/s^2

$$F_f = \frac{(p/S_u)S_u D d}{2} \qquad (17\text{–}47)$$

where (p/S_u) = specified life, from Fig. 17–21

S_u = ultimate tensile strength of the wires, psi

D = sheave or winch drum diameter, in

d = nominal wire rope size, in

Working Equations for Mine-Hoist Problem

$$F_b = \frac{E_r d_w A_m}{D} \qquad (17\text{–}48)$$

where E_r = Young's modulus for the wire rope, Table 17–24 or 17–27, psi

d_w = diameter of the wires, in

A_m = metal cross-sectional area, Table 17–24 or 17–28, in^2

D = sheave or winch drum diameter, in

$$n_s = \frac{F_u - F_b}{F_t} \qquad (17\text{–}49)$$

$$n_f = \frac{F_f - F_b}{F_t} \qquad (17\text{–}50)$$

Example 17–6

Given a 6 × 19 monitor steel ($S_u = 240$ kpsi) wire rope.
(a) Develop the expressions for rope tension F_t, fatigue tension F_f, equivalent bending tensions F_b, and fatigue factor of safety n_f for a 531.5-ft, 1-ton cage-and-load mine hoist with a starting acceleration of 2 ft/s^2 as depicted in Fig. 17–23. The sheave diameter is 72 in.
(b) Using the expressions developed in part (a), examine the variation in factor of safety n_f for various wire rope diameters d and number of supporting ropes m.

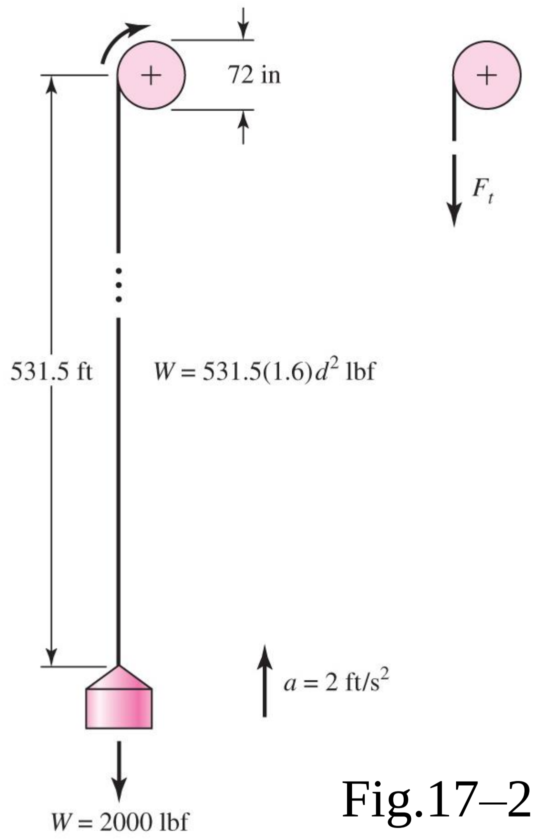


Fig.17–23

Example 17–6

(a) Rope tension F_t from Eq. (17–46) is given by

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right) = \left[\frac{2000}{m} + 1.60d^2(531.5)\right]\left(1 + \frac{2}{32.2}\right)$$

$$= \frac{2124}{m} + 903d^2 \text{ lbf}$$

From Fig. 17–21, use $p/S_u = 0.0014$. Fatigue tension F_f from Eq. (17–47) is given by

$$F_f = \frac{(p/S_u)S_u Dd}{2} = \frac{0.0014(240\,000)72d}{2} = 12\,096d \text{ lbf}$$

Equivalent bending tension F_b from Eq. (17–48) and Table 17–27 is given by

$$F_b = \frac{E_r d_w A_m}{D} = \frac{12(10^6)0.067d(0.40d^2)}{72} = 4467d^3 \text{ lbf}$$

Example 17–6

Factor of safety n_f in fatigue from Eq. (17–50) is given by

$$n_f = \frac{F_f - F_b}{F_t} = \frac{12\,096d - 4467d^3}{2124/m + 903d^2}$$

(*b*) Form a table as follows:

	n_f			
d	$m = 1$	$m = 2$	$m = 3$	$m = 4$
0.25	1.355	2.641	3.865	5.029
0.375	1.910	3.617	5.150	6.536
0.500	2.336	4.263	5.879	7.254
0.625	2.612	4.573	6.099	7.331
0.750	2.731	4.578	5.911	6.918
0.875	2.696	4.330	5.425	6.210
1.000	2.520	3.882	4.736	5.320

Example 17–6

Wire rope sizes are discrete, as is the number of supporting ropes. Note that for each m the factor of safety exhibits a maximum. Predictably the largest factor of safety increases with m. If the required factor of safety were to be 6, only three or four ropes could meet the requirement. The sizes are different: $\frac{5}{8}$-in ropes with three ropes or $\frac{3}{8}$-in ropes with four ropes. The costs include not only the wires, but the grooved winch drums.

Flexible Shaft Configurations

Fig.17–24*b*

Flexible Shaft Construction Details

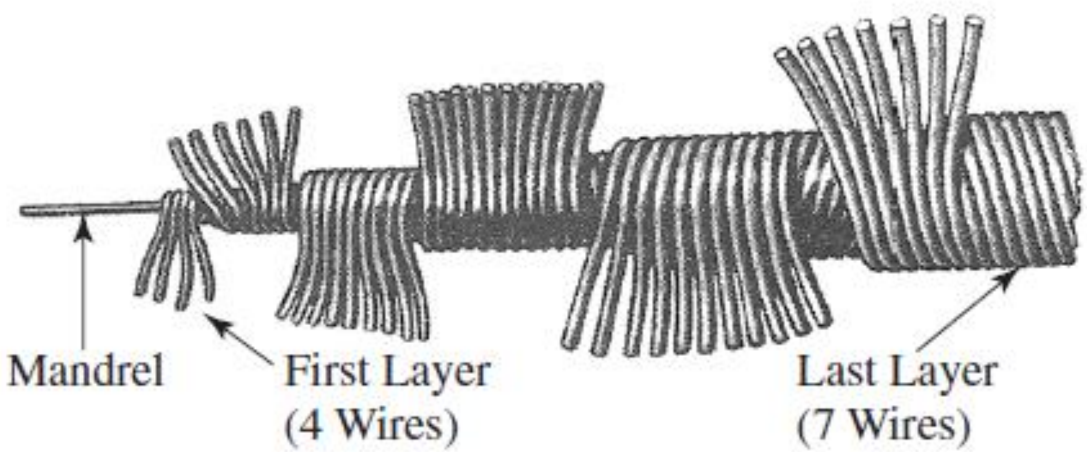


Fig.17–24*a*